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Dynamics of a slowly-varying sand bed in a circular pipe

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ABSTRACT

The long wave-length dynamics and stability of a bed of sand occupying the lower segment of a circular pipe are studied analytically up to first-order in the small parameter characterizing the slope of the bed. The bed is assumed to be at rest, with at most a thin sand layer (the bedload) moving at the sheared interface. When the sand bed is plane, with depth independent of position z along the axis of the pipe, the velocity of the liquid is known from previous studies of stratified laminar flow of two Newtonian liquids (the lower one with infinite viscosity representing the sand bed). When the depth of the sand bed varies with z, secondary flows develop in the cross-sectional (x, y) plane, and these are computed numerically, assuming that the sand bed remains a straight horizontal line in the cross-sectional plane. The mean shear stress acting on the perturbed sand bed is then determined both from the computed secondary flows and by means of the averaged equations of Luchini and Charru. The latter approach requires knowledge only of the flow over the unperturbed, flat sand bed, combined with an accurate approximation of the distribution of the perturbed stresses between the pipe wall and the sand bed. The perturbed stresses determined by the two methods agree well with each other. Using these stresses, it is then possible to apply standard theories of bed stability to determine the balance between the destabilizing effect of inertial (out-of-phase) stresses and the stabilizing effects of gravity and relaxation of the particle flux, and various examples are considered.

1. Introduction

The transport of sand/water slurries along a horizontal pipeline is of commercial importance, and has therefore been the subject of many studies, reviewed by e.g., Peker and Helvacı (2008); Goharzadeh et al. (2013); and Soepyan et al. (2014). The prediction and control of transport (or settling) of entrained sand in petroleum pipelines is similarly important (Salama, 2000).

At high fluid velocities the particles are suspended and flow with the fluid. However, at low velocities the particles (if denser than the fluid) sediment under gravity, and a stationary bed of particles forms on the lower side of the pipe (Turian et al., 1987). Our interest here lies in the regime of moderate fluid shear stress on the bed, when particles at the bed surface are slowly entrained into a thin moving layer (e.g., Oroskar and Turian, 1980; Takahashi and Masuyama, 1991; Doron and Barnea, 1995, 1996; Turian et al., 1987; Peysson et al., 2009). This moving layer (the bedload layer) has a thickness of just a few particle diameters.

Many studies have concentrated on the flow rates of the sand and water as functions of the applied pressure gradient (e.g., Doron et al., 1987; Kuru et al., 1995; Ouriemi et al., 2009a). However, a crucial issue for bedload transport is the shear stress exerted by the fluid flow over the bed: this stress determines the particle flow rate. The upper surface of the bed is usually wavy (rather than plane), so that the shear stress and particle flow rate are non-uniform in the streamwise direction, leading to the propagation of a complex pattern of sand waves, see e.g., the review by Charru et al. (2013). These waves are of both scientific and engineering interest: ripples and dunes are known to have strong consequences on flow rates and pressure gradients (Takahashi et al., 1989; Takahashi and Masuyama, 1991; Ouriemi et al., 2009b; Al-Lababidi et al., 2012).

The aim of this paper is to provide a set of area-averaged equations governing slow variations of the fluid flow and sand bed, consistent up to first-order in the small-slope parameter. We then use these equations to analyze the linear stability of the bed. The analysis is restricted to laminar flow, with the usual quasistatic assumption that the time scale for bed height variations is long compared to the hydrodynamic time scale, so that the flow may be calculated as if the bed profile were fixed.

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We first (Section 2) discuss the velocity profile and shear stresses in fluid flowing through a pipe in which the sand bed is uniform along the length of the pipe. When the height of the sand bed varies slowly in the axial direction, not only is there a slow variation in the axial velocity of fluid along the pipe, but secondary flows are set up in the cross-section. Such flows are discussed in Section 3. In Section 4 we review a standard theory for the movement of sand grains at the bed surface due to the hydrodynamic bed stress. In Section 5 we derive the set of area-averaged equations for the fluid flow rate, particle flow rate and bed height, assuming that the sand flux is a function of the mean stress averaged over the width of the bed (the detailed stress distribution is ignored). The equations are based on the analysis of Luchini and Charru (2010a) of slowly-varying laminar flows which appeals to the stationarity of the viscous dissipation term in the energy equation, combined with the approximation that the ratio of the shear force acting on the bed to the shear force acting on the wetted wall of the pipe is the same at first-order as at zeroth-order. These equations, although consistent up to the first-order in the small-slope parameter, require only the parallel-flow analytical results (*i.e.,* they do not require the calculation of the first-order flow disturbance over the slowly-varying sand bed). The validity of this analysis is confirmed by comparison with the full first-order numerical results presented in Section 3. As an illustration of the use of the area-integrated equations, a stability analysis of the plane bed is performed in Section 6.

The analysis is restricted to Newtonian fluids, and therefore is inappropriate for either concentrated slurries of particles or for non-Newtonian crude petroleum: however, it is a useful starting point even for such for fluids. The Reynolds number will be required to be sufficiently low for the basic flow within the pipe to be laminar, but, as is standard in long wavelength analysis of nearly parallel flow, the Reynolds number need not be small compared to unity (as will be discussed in Section 3). The regime that we shall investigate is that in which particles at the surface of the sand bed are just starting to move due to the stress imposed on them by the fluid flowing above them in the pipe. Thus the analysis applies to a restricted range of flow rates which is, nevertheless, an important one, since it separates the regime in which the bed is at rest (growing slowly if further particles are deposited) from that in which the particle bed starts to be eroded (as would be required for cleaning out the pipe). We shall re-visit these restrictions in Section 7, where they can be made explicit in terms of the analysis of Sections 2–6.

2. Liquid flow through a pipe with a uniform sand bed

The geometry that we consider is shown in Fig. 1. The pipe has radius R. A bed of sand at the base of the pipe subtends an angle $2\delta_b$ at the center of the pipe, and has a plane, horizontal upper surface AEC. The upper part of the pipe is occupied by liquid, and the portion of the circular pipe wall that is wetted by liquid subtends an angle $2\delta_w = 2(\pi - \delta_b)$ at the center of the pipe.

We set up Cartesian coordinates, with z axis parallel to the axis of the pipe and with (x, y) in the cross-sectional plane of the pipe. The y axis is vertical, along the symmetry axis, and the x axis is horizontal, joining the two triple points A and C where liquid, the pipe wall and the sand bed meet (Fig. 1). We assume that the interface between the sand bed and the liquid is plane, and that it coincides with the x axis $y = 0$. We shall occasionally use cylindrical polar coordinates (r, ψ, z), with $\psi = 0$ directed along the y axis.

The cross-sectional area A of the portion of pipe occupied by liquid can be found by elementary methods, and is

$$A = R^2\left(\delta_w - \tfrac{1}{2}\sin 2\delta_w\right). \tag{1}$$

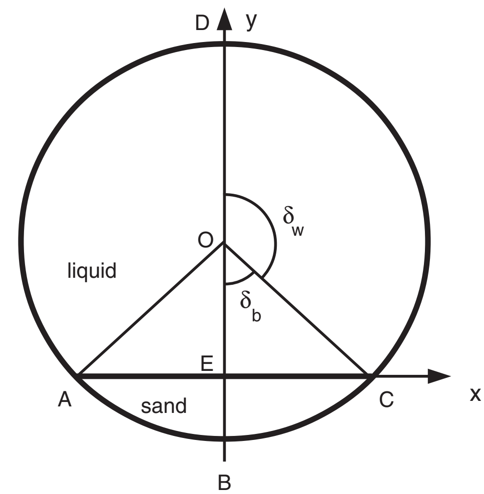

Fig. 1. Cross-section of the pipe, with sand at the bottom, and liquid above. The pipe has radius R and the maximum sand bed depth, EB, is h (3). The sand bed width AEC has length C_b, and the length of the wetted wall ADC is C_w (2).

In the cross-section, the length C_b of the sand bed, and the length C_w of the portion of the cylindrical wall wetted by liquid, are

$$C_b = 2R\sin\delta_b, \quad C_w = 2R\delta_w. \tag{2}$$

The maximum height of the sand bed, at $x = 0$, is

$$h = R(1 - \cos\delta_b) = R(1 + \cos\delta_w), \tag{3}$$

and we note for future use that

$$\frac{\partial A}{\partial h} = -C_b, \quad \frac{\partial C_b}{\partial h} = -2\cot\delta_w. \tag{4}$$

Particle velocities in the bedload layer are much smaller than the bulk fluid velocity, typically a fraction of the fluid velocity at a distance of one particle diameter above the bed at rest. Hence it is usual to calculate the fluid flow as if the wavy bottom were fixed (Charru et al., 2013), and the errors introduced by this approximation are small. The liquid therefore satisfies a no-slip boundary condition both at the bed/liquid interface and on the circular wall of the pipe.

Flow of two fluids in such a geometry has been well studied (Bentwich, 1964; Ranger and Davis, 1979; Brauner et al., 1996; Biberg and Halvorsen, 2000), because of its importance when pumping two fluids that have separated due to their density difference. If the viscosity of the lower fluid is taken to be infinite, this lower fluid becomes stationary, and the flow of the upper fluid corresponds to fluid flowing above a sand bed. We present a short summary of the analysis and analytic predictions for this case of a uniform flat bed in Appendix A. However, we shall eventually need to use numerical methods, and it is convenient to do so even for the simplest case of a uniform sand bed. The analytic results then provide a useful check on the accuracy of the numerical scheme.

The liquid is assumed to be Newtonian and incompressible, with density ρ and viscosity η. If the bed of sand is uniform, the liquid velocity w in the z direction satisfies

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)w = -G/\eta, \tag{5}$$

where $-G < 0$ is the axial pressure gradient. We solved the Poisson Eq. (5), subject to a no slip condition at the boundaries, by means of the finite element package FreeFem++ (Hecht, 2012). By way of example, Fig. 2 shows isolines of the velocity $w(x, y)$, normalized by Q/R^2 where Q is the volumetric flow rate, for the case $h/R = 0.5$. Note that the maximum velocity is greater than the value $2/\pi$ for $h/R = 0$, as expected.