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# Multiple holdup solutions in laminar stratified flow in inclined channels

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ABSTRACT

This study exposed results of investigations on the two-fluid laminar–laminar stratified flow in an inclined channel. Multi-holdup regions have been mapped and visualized in a new way by plotting solution surface of the holdup equation. This representation clearly exhibits the link between backflow situation and multiple holdup solution occurence. The multiple solution problem can be addressed following different approaches. First, the solutions of the two-fluid system are interpreted in term of the minimization of a potential function. Secondly, the minimization of the dissipation rate at the feasible holdup is investigated and thirdly, a long-wave stability analysis is considered.



## 1. Introduction

The stratified flow model presented by Taitel and Dukler (1976b) is a one-dimensional, two-fluid mechanistic model widely used to describe the transport of multiphase flows in chemical process, nuclear and petroleum industries. This model assumes that two fluids are flowing in two separate continuous layers without mass transfer between the different phases and in steady, isothermal conditions. The two parameters of interest in the design of such systems are the pressure drop and the holdup. Baker et al. (1988) pointed out that the model can predict as many as three solutions of the holdup for some inclined systems, which unluckily often correspond to operating conditions. Landman (1991) dedicated theoretical investigations on this issue and showed that the occurence of multiple values for holdup and pressure drop persists when considering the exact known solutions for laminar–laminar flow in a channel.

Barnea and Taitel (1992, 1994a,b,c) carried out stability studies and then recommended that both structural and interfacial Kelvin–Helmholtz stability analyses must be performed to determine which holdup solution actually occurs. For upwardly inclined gas–liquid flow, the three solutions correspond to three liquid fraction or liquid holdup. The thinest and the thickest holdup steady-state solutions are accepted to be linearly stable with respect to the structure, while the intermediate solution is unstable and thus not realizable. Performing non-linear structural stability analysis, the thickest solution appears to be unstable in response to finite disturbances. This instability was attributed to severe oscillations before reaching the steady-state solution eventually passing through negative net liquid flow rate. As a result, when a single rather thick liquid holdup solution exists, this one is necessarily linearly stable but may be unstable to finite disturbances at the same time as observed by Barnea and Taitel (1992). These authors stated that in this situation the smooth stratified flow pattern would be replaced by another flow structure (waves, slug or annular flow for instance). Then before validating a solution, a complementary Kelvin–Helmholtz analysis focusing on the interface stability must be performed in order to verify the validity of the smooth stratified flow pattern assumption. According to the authors, when multiplicity of the solutions is expected only the thinest liquid holdup must be realizable.

To be noted that these results (Barnea and Taitel, 1992, 1994a,b,c) were obtained using wall shear stress closure laws derived from the single-phase flow configuration. Unfortunately, this kind of relation is not able to capture situation where local backflow occurs near the wall while the net flow rates remain positive. Biberg (1999, 2002) and Biberg and Halvorsen (2000) examined the exact pipe and duct flow laminar solutions and put the possibility of such situation forward. Looking at velocity profiles, Ullmann et al. (2004) observed that backflow configurations are usually linked to the intermediate and the upper liquid holdup solution. Necessity of improving the usual single-phase-based closure laws by integrating the interaction between the phases was thus pointed out. Recently, long wave stability analysis using exact

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laminar channel flow solutions has been performed by Kushnir et al. (2014) on the specific issue of multiple solutions. It was suggested the existence of a region where the three solutions are stable to long wave disturbances and that the intermediate solution remain stable in the whole multiple solutions region. Eventually, for regions where only one steady-state holdup is expected, this last one may also be unstable as proposed by Barnea and Taitel (1992, 1994a,b,c). It is worth noting that both the Kelvin–Helmholtz and the long waves interfacial stability analyses aim at validate the existence of a smooth stratified flow pattern. However, when multiple solutions are predicted for a smooth stratified pattern, these analyses do not elucidate the question of the solution to pick out.

In the present study, the multiple holdup occurence was investigated focusing on the backflow situation for the exact laminar channel flow solutions. The steady-state holdup solutions were represented as a response surface allowing the characterization of these solutions in relation with physical considerations. Three different approaches are reviewed to analyse the solution to pick out. Firstly, a new approach is proposed by the mean of the catastrophe theory. This theory provides tools for the interpretation of the occurence of the multiple holdup solutions and their stability. Secondly, the principle of minimization of dissipation is also considered as a way to predict the most feasible solution. Finally, following Kushnir et al. (2014) a long-wave stability analysis is performed and results were considered for the selection of the holdup solution.

## 2. Modelling and holdup equation

In the stratified flow model presented by Taitel and Dukler (1976b), the flow was supposed to be steady, fully-developed, isothermal and unidirectional with no mass transfer between the different phases. In Fig. 1 is described the configuration where two fluids, a light one and an heavier one (denoted respectively $L$ and $H$), are flowing into a pipe inclined by the angle $\theta$. For stratified flow, the integral form of the continuity equations for the two phases are:

$$\frac{\partial}{\partial t}(\rho_H A_H) + \frac{\partial}{\partial x}(\rho_H A_H U_H) = 0 \tag{1}$$

$$\frac{\partial}{\partial t}(\rho_L A_L) + \frac{\partial}{\partial x}(\rho_L A_L U_L) = 0 \tag{2}$$

and the momentum equations are:

$$\frac{\partial}{\partial t}(\rho_H A_H U_H) + \frac{\partial}{\partial x}(\rho_H A_H \gamma_H U_H^2) = -\tau_H S_H - \tau_{iH} S_i - \rho_H A_H g \sin(\theta) - \frac{\partial A_H P_H}{\partial x} + P_{i,H}\frac{\partial A_H}{\partial x} \tag{3}$$

$$\frac{\partial}{\partial t}(\rho_L A_L U_L) + \frac{\partial}{\partial x}(\rho_L A_L \gamma_L U_L^2) = -\tau_L S_L - \tau_{iL} S_i - \rho_L A_L g \sin(\theta) - \frac{\partial A_L P_L}{\partial x} + P_{i,L}\frac{\partial A_L}{\partial x} \tag{4}$$

where $h$ is the level of the heavy fluid, $A_j$ is the cross-sectional area of the phase $j$ ($j = L, H$), $U_j$ its mean velocity, $\gamma_j$ the velocity shape factors, $S_j$ the wall perimeter, $\rho_j$ the density, $\tau_j$ and respectively $\tau_{ij}$ the shear stresses exerted on the fluid $j$ by the wall and respectively the interface, and $S_i$ is the interfacial perimeter. $P_j$ is the mean pressure in the section $A_j$ and $P_{i,j}$ the pressure at interface, which could be different in each phase due to surface tension effects.

The effect of surface tension is neglected ($P_{i,H} = P_{i,L}$) and the average pressure gradient $\partial(A_j P_j)/\partial x$ is usually evaluated assuming an hydrostatic evolution of the pressure in the section $A_j$. This assumption is reasonably adopted here, although the impact on the separated flow models is discussed in the literature (see Jones and Prosperetti (1985) for instance). Given that the interfacial shear stresses are linked by $\tau_i = -\tau_{iH} = \tau_{iL}$, utilizing Eq. (1) and (2) and subtracting Eq. (4)/$A_L$ to Eq. (3)/$A_H$ yields:

$$\rho_H\frac{\partial U_H}{\partial t} - \rho_L\frac{\partial U_L}{\partial t} + \left[\rho_H(1-\gamma_H)\frac{U_H}{A_H} + \rho_L(1-\gamma_L)\frac{U_L}{A_L}\right]\frac{\partial A_H}{\partial t} + \rho_H U_H\frac{\partial \gamma_H U_H}{\partial x} - \rho_L U_L\frac{\partial \gamma_L U_L}{\partial x} + (\rho_H - \rho_L) g\cos(\theta)\frac{\partial h}{\partial x} = F \tag{5}$$

where

$$F = \tau_L\frac{S_L}{A_L} - \tau_H\frac{S_H}{A_H} + \tau_i S_i\left(\frac{1}{A_H} + \frac{1}{A_L}\right) - (\rho_H - \rho_L) g \sin(\theta) \tag{6}$$

To remain valid, the formulation of the term $F$ must include the expression of the shear stresses evaluated in the general case and not only at fully-developed steady-state solutions. The fully developed steady-state solutions are characterized by the condition $F = 0$ and the shear stresses in this case can be analytically expressed. At this stage, indeed, the common procedure is to use closure laws in order to calculate $\tau_H, \tau_L$ and $\tau_i$. In the past (Taitel and Dukler, 1976b), these relations were first derived by analogy with single-phase flow and many studies focused on improving the precision of such closure laws for real configurations (Ng et al., 2002, 2004 and Hanratty, 2013 for instance). Despite of being more academic than practical, the simple case of laminar–laminar flow between two infinite parallel plates separated by distance $D$ provides an exact analytical expression of the velocity profiles when the fully developed steady state flow is concerned. Thus, there is no need to use approximative closure relations. In that case, using the continuity of the velocity and the shear stress at the interface and integrating the velocity profiles on both the phases, one may derive the following expressions for the wall shear stresses at the steady-state conditions:

$$\tau_H = \mu_H\frac{\partial u_H}{\partial y}\bigg|_{y=0} = \underbrace{3\frac{\mu_H\langle U_H\rangle}{\varepsilon D}}_{free\ surface\ flow} - \underbrace{3\frac{\mu_H\mu_L}{\mu_H/\varepsilon + \mu_L/(1-\varepsilon)}\frac{\langle U_L\rangle - \langle U_H\rangle}{(1-\varepsilon)\varepsilon D}}_{shear\ flow} \tag{7}$$

and

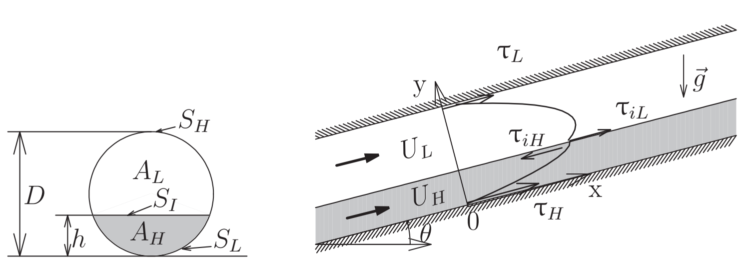


**Fig. 1.** Scheme of the stratified flow configuration.