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A gas entrainment model for hydraulic jumps in near horizontal pipes

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ABSTRACT

We develop a simplified physical model for gas entrainment in the hydraulic jump, where the subcritical flow fills the pipe diameter. The model is compared to experimental data obtained in a previous study where the entrained gas flux (air) is measured directly. Different pipe diameters and fluids were considered. Based on the structure of the hydraulic jump, we suggest that an important mechanism is entrainment by liquid that is expelled from the front and plunges into the incoming liquid ahead of the front, coupled with gas leakage out of the front. Turbulence generation and circulation behind the front are accounted for.

The model performs well when the entrainment parameters are tuned to values reported elsewhere for "plunging" liquid jets. With a single set of entrainment constants, we obtain satisfactory results for the different inflow velocities and pipe diameters. The model is designed for highly turbulent flows, where the effect of fluid viscosity is minor or absent. The entrainment rate could not be linked in a simple way to the Froude number. The model constitutes an explicit algebraic relation between the entrainment rate and the flow parameters (average inflow velocity and height of the jump).

1. Introduction

Many physical properties of the classical hydraulic jump with free surfaces are now well understood (e.g., Hager and Bremen, 1989; Chanson, 2009). Hager et al. (1990) found that the recirculation length of the roller depends only on the Froude number for moderate Froude numbers (below approximately 8). For higher Froude numbers, the effects of inflow Reynolds number and inflow aspect ratio become significant. Hager and Bremen (1989) studied the effects of wall friction and it was shown that the liquid depth ratio is always smaller than the frictionless approximation described by the Bélanger equation, and that it is dependent on the inflow Reynolds number and the inflow aspect ratio. For the free surface hydraulic jump in circular pipes, Stahl and Hager (1999) also found a Froude number dependent recirculation length of the roller. Dyment (1998) established jump conditions for both free and confined hydraulic jumps in pipes or closed conduits of a more general shape. Recently, Ma et al. (2010) summarized the experimental work of a number of different authors for free surface hydraulic jumps and liquid jets, and proposed a common scaling law for the air entrainment rate.

The current study concerns the gas entrainment in a hydraulic jump in a closed pipe when the subcritical flow fills the pipe diameter. The jump can be made stationary in the lab frame, and the volume flux of gas through the jump can be then be measured accurately. The model development in the current work is based on such gas flow measurements and on qualitative observations of the front structure of the jump, circulation pattern in the mixing zone (or roller), and the bubble distribution.

The gas entrainment in hydraulic jumps in pipes (for a diameter-filling subcritical flow), has been studied experimentally by e.g., Jepson (1987), Zhou and Jepson (1993), Julshamn et al. (2004), Julshamn (2006), Pan (2009), Mortensen et al. (2011), and Pothof and Clemens (2011). Mortensen et al. (2011) found that the pipe diameter did not affect the relative entrainment rate (air discharge or volume flux, normalized to the water discharge). Unexpectedly, they also found that higher temperature reduces the relative entrainment rate, and increased the apparent bubble sizes in the aerated mixing zone. Pothof and Clemens (2011) studied air–water flow in downward sloping pipes and obtained correlations for the air discharge for the plug flow and blow back flow regimes.

For slug flow, the hydraulic jump moves with significant velocity relative to the pipe walls and the flow pattern in the recirculation zone is therefore altered. A double circulation pattern may develop with backflow (relative to the propagating front) near the pipe wall and near the base of the jump, and forward flow in the central region of the recirculation zone. In contrast, the stationary hydraulic jump has one roller only. Jepson (1987) still argued that the basic hydrodynamic mechanisms and aeration processes are similar for the stationary hydraulic jump and the slug flow.

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Guet et al. (2006) summarized the slug flow work by Brauner and Ullmann (2004) and Nydal and Andreussi (1991), and classified the modeling work into two categories: (a) gas entrainment due to turbulence in the incoming (subcritical) flow and (b) gas entrainment due to the pressure jump over the slug front.

A common factor characterizing earlier entrainment correlations for both hydraulic jumps and slugs, is the dependency on the Froude number (in terms of the relative velocity between the front and liquid layer). However, simple relations in terms of the Froude number do not capture the dependency on fluid properties, such as the surface tension, liquid/gas densities and viscosities. Furthermore, we would expect that the net entrainment rate depends on gas coming in at the foot of the jump, entrainment over the front area, and advective/turbulent transport upstream of bubbles out of the recirculation zone. That the Froude number alone is enough to characterize these different entrainment processes is doubtful.

Chanson and Murzyn (2008) pointed out that at least 10 non-dimensional numbers (including the Reynolds and Morton numbers that reflect the viscosities and surface tension in the system) can potentially characterize the air entrainment in the hydraulic jump, and the Froude number is only one of them. Indeed, Chanson and Murzyn pointed out significant scale-dependent effects for smaller hydraulic jumps, and that dynamic similarity of two phase flows in hydraulic jumps cannot be achieved with a simple Froude number similitude. Bonizzi and Issa (2003) suggested that the gas entrainment process for slugs and hydraulic jumps are similar, and adopted a Froude-number-based entrainment relation for the hydraulic jump due to Chanson (1996) in their slug flow model. It was stressed that this approach did not account for viscosity and interfacial tension, and further research was encouraged.

Later, Brauner and Ullmann (2004) made an analysis for gas entrainment from Taylor bubbles, as an analogy to slug flow entrainment, rather than resorting entirely to empirical correlations in terms of the Froude number. The model by Brauner and Ullman assumes a balance between the rate of turbulent kinetic energy production and the rate of bubble surface energy production (in terms of the surface tension). Zhang et al. (2003) modeled the liquid holdup in the slug (or equivalently the void fraction) by assuming that the total surface free energy of the bubbles is proportional to the total turbulent kinetic energy in the slug. One should however expect that the surface energy of the bubbles is only a small fraction of the turbulent kinetic energy for reasonable values of the surface tension (and assuming typical pipe flow Reynolds numbers).

The qualitative impression from the experiments of Julshamn (2006) on the entrainment in hydraulic jumps in pipes suggests a similarity to entrainment due to a jet falling into a stationary pool of liquid (Kockx et al., 2005), or to the entrainment due to rolling, aerated liquid at the front of breaking surface gravity waves (e.g., Longuet-Higgins and Turner, 1974). The latter is referred to as the "spilling breaker" or "plunging breaker" in oceanography. At moderate to large inflow velocities, the front of the hydraulic jump is observed to "gallop" upstream with liquid being expelled and impacting the incoming liquid interface (Julshamn, 2006). The gas is sometimes entrained intermittently, with the generation of easily visible bubble plumes, similar to those generated by breaking ocean waves.

The present work develops a simplified model in terms of entrainment by liquid expelled from the front, as it impacts the incoming liquid ahead of the front, and in terms of leakage of gas out of the front by advection and turbulent diffusion. The turbulent kinetic energy and larger scale circulation in the mixing zone flow provide a dynamic pressure that can overcome the interfacial energy barrier (in terms of surface tension and gravity) so that liquid can be ejected. A simple energy balance determines the impact velocity (of the ejected liquid) into the liquid layer, and we adopt a scaling law for the associated gas entrainment rate (Brattberg and Chanson, 1998; Ma et al., 2010). The gas leakage out of the front by advection and turbulent diffusion of bubbles provides a saturation effect such that the void fraction in the recirculation zone is always limited (and guaranteed to be less than unity), regardless of the velocity (or Froude number) of the incoming liquid.

First, we present a simple turbulence model for the hydraulic jump that accounts for the general momentum and energy balances (including the change in gravitational potential energy over the front). Then, we present the ingredients of the entrainment model, before we compare to measured entrainment rates. We compare with data for four different fluids with different viscosities and surface tensions, in combination with air at atmospheric pressure, in two different pipe diameters.

2. Entrainment model

We will consider the following contributions to the net entrainment flux:

- Entrainment of gas by expelled liquid plunging into the upstream liquid layer, denoted Φ_p. The upstream liquid is a supercritical, free surface "open channel" flow.
- Leakage of gas upstream through the front, Φ^-.
- Gas "entrapment" the base of the front where the local shear is large, Φ_f.

These contributions are illustrated in Figs. 1 and 2. The total gas entrainment flux [in units of (m^3/s)] is then simply the sum of the contributions,

$$\Phi = \Phi_p - \Phi^- + \Phi_f. \tag{1}$$

We obtain scaling formulae for the individual fluxes below.

Fig. 3 summarizes the velocities associated with the entrainment, and defines the extent of the mixing zone behind the front, where there is a large void fraction. The mixing zone void fraction α_{mix} will be an intrinsic part of the entrainment model. The turbulent kinetic energy and larger scale circulation behind the front are important quantities for Φ_p and Φ^-, and we will therefore consider the associated turbulence and circulation sub-models to begin with.

2.1. Turbulence generation in the hydraulic jump

Turbulence leads to dispersion of bubbles upstream and downstream in the mixing zone as well as expulsion of liquid at the front. By considering the balances of momentum and mechanical energy over the hydraulic jump, one can obtain an equation for the average turbulent kinetic energy in the mixing zone. The turbulent kinetic energy is here somewhat higher than in the fully developed flow that may be realized further along the pipe.

The axial energy flux density (J/m^2/s) at any point in the flow is

$$F = v(p + 1/2\rho v^2 + \rho g y), \tag{2}$$

where v is the axial velocity (the component parallel to the pipe walls), p is the pressure, ρ is the mass density, g is gravity, and y is the wall normal coordinate. We combine the mechanical energy equation with the momentum equation, and perform suitable control volume computations to obtain the jump conditions. We then obtain the following energy contributions that are available for generation of turbulent kinetic energy in the mixing zone,

$$\Delta E_e = A|U_{mix}|\left[\frac{1}{2}\rho_L(U_{mix} - U_{LB})^2 - g_\perp(A_s/A)(\rho_l - \rho_G)(\bar{y}_{GB} - h)\right], \tag{3}$$

$$\Delta E_0 = |f_{AB}U_{mix}|. \tag{4}$$