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Brief communication

In-line interaction between two spherical particles due to a laminar wake effect

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1. Introduction

The description of the motion of single bodies in an unbounded fluid is very relevant because it is the starting point to the understanding of particulate two-phase flow dynamics. In some cases, the two-phase flow modeling can be carried out considering that the interfacial forces can be approximated by scaling the drag of constituent dispersed bodies considering them isolated. This can be true for very-diluted systems. However, the more the volumetric fraction of the particulate phase is increased, the closer its elements will be, and thus they can interact. Also, the forces on an interactive element of the dispersed phase are strongly dependent on the instantaneous flow field around it. In this instance, the simplified description of diluted two-phase flows is not sufficient in order to predict the behavior of more dense systems.

Let us consider dense solid–fluid two-phase flow systems. If two particles are moving close to each other, then the flow structure around each one is quite different from that of an isolated particle. The flow falling on each particle contains non-homogeneities resulting from the flow disturbance induced by the other body. Thus, the drag force, which is dependent on the interfacial integration of total stresses, is different from that of an isolated particle. Moreover, the hydrodynamic force includes other force contributions due to flow non-uniformity (inertial forces), such as flow acceleration and added mass forces (Taylor, 1928; Maxey and Riley, 1983). The understanding of particle–particle interactions and their quantification is very important for adequate two-phase flow modeling, because this can be the basis to construct interfacial force closures for two-phase models for fluid–solid equipments.

Since $Re < 200$ for many industrial applications in two-phase flows – being Re the Reynolds number defined through the particle diameter and its relative velocity, with respect to the local fluid velocity – the analysis of particle–particle interaction in the intermediate flow regime is very important (Zhu et al., 1994; Chen and Lu, 1999). Aware of this, several researchers have studied the variation of the hydrodynamic force of two fixed spherical particles placed at different relative positions from each other. In this regard, experimental and numerical works have been carried out.

Some of the pioneering experimental works were conducted by Rowe and Henwood (1961) and Tsuji et al. (1982) for in-line interaction. By a pendulum method, they measured the force exerted by a moving fluid on interactive rigid spheres, and they observed that this force is smaller than that exerted on an isolated sphere. Their experiments showed a general trend for this type of interaction if $Re \sim O(100)$. Subsequently, Zhu et al. (1994) conducted measurements using a micro-force balance system for Re ranging from 20 to 130. They recognized a significant dependence of the hydrodynamic force on the trailing body with Re and the inter-particle distance. Chen and Lu (1999) and Chen and Wu (2000) extended the experimental analysis varying the relative position of a sphere around the other as a reference particle.

Even if the in-line interaction is a specific case, it continues to be useful to understand the theoretical basis of hydrodynamic interactions among bodies. In this regard, the in-line interaction results of particular interest since the wake of the leading particle directly affects the drag and inertial forces experienced by the trailing particle, and the interactions are maximized.

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Consequently, relating the wake flow structure with the hydrodynamic force is a fundamental step to appropriately understand this type of interaction.

In general, for in-line interaction, the increase of the inter-particle distance is related to an increase in the hydrodynamic force on the trailing particle (Zhu et al., 1994). This trend has been observed in numerical works, too (Tal et al., 1984; Tsuji et al., 2003; Prahl et al., 2007). However, while numerical works predict that an increase in *Re* causes a reduction in hydrodynamic force on the trailing particle, this behavior is not evident in experimental works (Zhu et al., 1994; Tsuji et al., 2003; Prahl et al., 2007).

Zhang and Fan (2002) and Ramírez-Muñoz et al. (2007) proposed models in order to predict the total drag and the hydrodynamic force on an interactive trailing particle. They attempted to have as foundation the knowledge of the wake flow structure behind the leading particle. For this purpose, they incorporated the well-known far wake velocity profile behind an axisymmetric body in their expressions (Batchelor, 1967). However, this approximation was obtained under idealized assumptions, not necessarily valid for their conditions of study, even more so taking into account that the hydrodynamic interaction is more significant at separation distances of the magnitude order of one particle diameter, in a region that do not corresponds to the far wake. Moreover, in both works they supposed that the drag coefficient is inversely proportional to *Re*, which is only true for the creeping flow regime.

In order to improve the comprehension of these phenomena, the flow structure of an axisymmetric laminar wake behind a spherical particle was obtained via numerical simulations. By incorporating an artificial origin in the far wake velocity solution – which was fitted to numerical data – an equation for the axial velocity profile was proposed. Furthermore, the relationship between the wake flow and the hydrodynamic force on the trailing particle at inter-particle distances of the magnitude order of one particle diameter was analyzed. Given the above, a model to predict this force was proposed, and the relevance of inertial forces resulting from wake flow non-homogeneities was discussed.

2. Hydrodynamic force on the trailing particle

Let us assume that two spherical particles are aligned with the direction of a Newtonian and uniform flow. Both bodies have the same diameter and are fixed in space. Then, a coordinate framework can be fixed at the rear of the leading particle. This system, with axial symmetry, is schematically shown in Fig. 1.

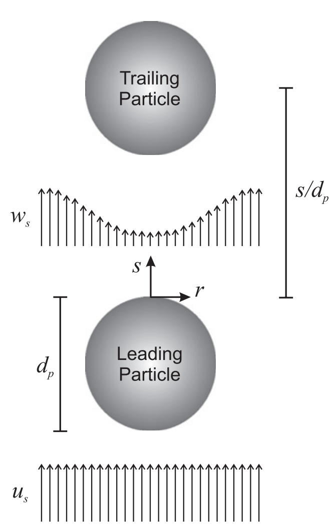

Fig. 1. In-line interaction between two spherical particles.

The dimensionless vertical component of the hydrodynamic force on the trailing particle can be expressed as follows (Ramírez-Muñoz et al., 2007)

$$\frac{F_{HD}}{F_{D1}} = \frac{F_{D2}}{F_{D1}} + \frac{F_I}{F_{D1}}, \tag{1}$$

where F_{HD} is the total hydrodynamic force. F_{D1} represents the steady drag of the leading sphere (sub-index 1) and F_{D2} the quasi-steady drag on the trailing sphere (sub-index 2). Also, F_I is the convective inertial force due to axial non-uniformity of the wake flow, which includes two contributions: the acceleration fluid force and the added mass force (Taylor, 1928).

Now, let us consider the equations for the drag of both spheres, respectively

$$F_{D2} = C_{D2}\frac{\pi}{4}d_p^2\frac{1}{2}\rho\bar{w}_s^2, \tag{2}$$

$$F_{D1} = C_{D1}\frac{\pi}{4}d_p^2\frac{1}{2}\rho u_s^2, \tag{3}$$

and their ratio

$$\frac{F_{D2}}{F_{D1}} = \frac{C_{D2}}{C_{D1}}\bar{W}^2. \tag{4}$$

Here

$$\bar{W} = \frac{\bar{w}_s}{u_s}. \tag{5}$$

In above equations, C_{D2} and C_{D1} are the drag coefficients of the trailing and leading spheres, respectively; d_p is the particle diameter and ρ is the fluid density. Meanwhile $\bar{w}_s$ is a characteristic velocity in the wake which is to be used in order to estimate the quasi-steady drag of the trailing particle and u_s is the uniform flow velocity. The drag coefficients ratio can be expressed as

$$\frac{C_{D2}}{C_{D1}} = \frac{Re_1}{Re_2}\beta, \tag{6}$$

where $Re_1 = \rho u_s d_p/\mu$ and $Re_2 = \rho\bar{w}_s d_p/\mu$ are the Reynolds numbers of the leading and the trailing particle, respectively, and μ is the viscosity of the fluid. Furthermore, β provides a correction factor for the main term of O(Re^{-1}) from the drag coefficient expressions. By using the Clift et al. (1978) proposed equation for the drag of rigid spheres, we have

$$\beta = \frac{1 + 0.1935Re_2^{0.6305}}{1 + 0.1935Re_1^{0.6305}}. \tag{7}$$

The inertial forces contribution in Eq. (1) may be expressed as follows (Ramírez-Muñoz et al., 2007):

$$F_I = m_f(1 + C_M)\bar{w}_s\frac{d\bar{w}_s}{ds}, \tag{8}$$

where m_f is the mass of fluid displaced by the trailing sphere and C_M = 0.5 is the added mass coefficient for a spherical body. By dividing Eq. (8) by Eq. (3), and substituting the resultant expression in Eq. (1) together with Eqs. (4) and (6), we have

$$\frac{F_{HD}}{F_{D1}} = \bar{W}\left(\beta + \frac{2d_p}{C_{D1}}\frac{d\bar{W}}{ds}\right). \tag{9}$$

Yuan and Prosperetti (1994) suggested that one way to estimate the drag of a trailing body could be using the same drag coefficient expression than for a leading body, but with a relative velocity reduced by the wake effect. Following this line of thinking, it is reasonable to propose the use of an average of the wake velocity profile as the characteristic velocity into Eq. (5) (Ramírez-Muñoz et al.,