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Brief communication

Three-dimensional convective and absolute instabilities in pressure-driven two-layer channel flow

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1. Introduction

The stability of two-layer flows in planar channels and pipes has received considerable attention in the literature experimentally, theoretically and numerically. This is due to the central importance of these flows to numerous engineering applications, such as the cleaning of fast-moving consumer good plants, transportation of crude oil in pipelines (Joseph et al., 1997), mixing of liquids using centerline injectors, up-stream of static mixers (Cao et al., 2003), and the removal of highly viscous or elasto-viscoplastic material adhering to pipes by using fast-flowing water streams (Regner et al., 2007).

The instability of two-dimensional disturbances in two-fluid Poiseuille flows has been studied by many authors via linear stability analyses (Yiantsios and Higgins, 1988b; Hooper and Boyd, 1983; South and Hooper, 1999; Frigaard, 2001), with some being carried out in the long-wave limit (Yih, 1967; Yiantsios and Higgins, 1988a; Khomami, 1990a,b), as well as experimental techniques (Kao and Park, 1972). An extended review can be found in (Boomkamp and Miesen, 1996). Sahu et al. (2007) studied the linear instability of two-dimensional disturbances in a pressure-driven two-layer channel flow, wherein a Newtonian fluid layer overlies a layer of a Herschel–Bulkley fluid. Their results indicate that increasing the yield stress, prior to the formation of unyielded zones, and shear-thickening tendency are destablising. The convective and absolute nature of two-dimensional disturbances in a similar system is studied by Valluri et al. (2010). The stability maps demarcating the areas of absolute and convective instabilities as a function of other parameter values were presented. Frigaard (2001) studied the two-dimensional linear stability of two-layer Poiseuille flow of two Bingham fluids. Unlike the study of Sahu et al. (2007), the case studied by Frigaard (2001), involves an unyielded region between the Newtonian fluid and the yielded part of the Bingham fluid. Interfacial waves would not develop under such conditions; this suppression of interfacial modes then leads to super-stable two-layer flows (Frigaard, 2001). On the other hand, the effect of three-dimensional disturbances on the stability of pressure-driven channel flow, has received little attention.

Squire (1933) studied the stability of viscous fluid flow between parallel walls and found that every unstable three-dimensional disturbance is associated with a more unstable two-dimensional disturbance at a lower value of the Reynolds number. This result is commonly known as ‘Squire’s theorem’ (Drazin and Reid, 1985) and the connection between the two- and three- dimensional disturbances is known as ‘Squire’s transformation’. For two superposed fluids in plane Poiseuille flow, Yiantsios and Higgins (1988a) showed that three-dimensional disturbances are associated with smaller Reynolds numbers, and larger capillary contributions and density stratifications. The larger capillary contributions are stabilising for all parameter values, as is density stratification provided the density of the upper fluid is lower than that of the lower one. Thus, although a Squire’s transformation can exist for all flow parameters, a Squire’s theorem can only exist provided the Reynolds

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number has a destabilising effect. They found that in the absence of surface tension and gravitational effects, Squire's theorem is valid for $t_r > \sqrt{m}$ since the Reynolds number is destabilising; here, t_r is the thickness ratio of the two fluids and m is the viscosity ratio. For $t_r < \sqrt{m}$, the Reynolds number is stabilising and Squire's theorem no longer exists. This is also true in the presence of destabilising density stratification.

Following the remark by Yiantsios and Higgins (1988a), in the absence of a Squire's theorem it is necessary to perform a three-dimensional linear stability analysis in order to determine whether or not two-dimensional disturbances correspond to the most dangerous ones. Sahu and Hema (2011) examined the three-dimensional linear stability of two-layer plane Poiseuille flow, wherein a Newtonian fluid layer overlies a layer of a Herschel–Bulkley fluid, focussing on the range of parameters for which Squire's theorem does not exist. They demonstrated through a linear stability analysis the presence of three-dimensional instabilities. Malik and Hooper (2007) also studied the effect of three-dimensional disturbances on two-fluid channel flow, wherein both the fluids are Newtonian. Using an energy analysis they showed that maximum amplification of the disturbances is due to the "lift-up effect" as in case of single phase flow. They also found that, for some parametric regime, the maximum disturbance energy growth is associated with three-dimensional disturbances.

In this paper, a generalized linear stability analysis (Huerre and Monkewitz, 1990; Chomaz, 2005; Schmid and Henningson, 2001) (in which both the spatial wavenumber and temporal frequency are complex) of three-dimensional disturbances is carried out, which allows the demarcation of the boundaries between convectively and absolutely unstable flows in the space of relevant parameters: the Reynolds number and a viscosity ratio. To the best of our knowledge, this type of analysis, which has been performed previously for jets, mixing layers, wakes, boundary layers etc. for two-dimensional disturbances, has not been carried out for two-fluid channel flows in the context of three-dimensional disturbances.

The rest of this paper is organised as follows. Details of the problem formulation are provided in Section 2, and the results of the linear stability analysis are presented in Section 3. Concluding remarks are provided in Section 4.

2. Formulation

A pressure-driven channel flow of two immiscible Newtonian and incompressible fluids is considered. A rectangular coordinate system, (x,y,z), is used to model this flow (U_j,V_j,W_j), where x, y and z denote the streamwise, spanwise and wall normal coordinates, respectively, as shown in Fig. 1. U_j, V_j and W_j are the velocity components of fluid 'j' in the streamwise, spanwise and wall normal directions, respectively. The rigid and impermeable channel walls are located at $z = 0$ and $z = H$, respectively, and the sharp interface, which separates the immiscible fluids, is at $z = h^0$. The height of the channel, H, and $U_m \equiv Q/H$ are used as the length and velocity scales, respectively, in order to nondimensionalise the equations of motion, where Q denotes the total flow rate per unit transverse length. The viscosity and density have been scaled with μ_2 and ρ_2, respectively, such that the viscosity ratio, $m \equiv \mu_1/\mu_2$, and density ratio, $r \equiv \rho_1/\rho_2$, wherein μ_1 and ρ_1, μ_2 and ρ_2 are the viscosity and the density of the lower and upper fluids, respectively. The reduced dimensionless pressure P_j in fluid 'j' is related to the corresponding total dimensional pressure p_j through

$$P_j = \frac{H}{\mu_2 U_m}\left[p_j + \rho_j g(z-h)\right] \ (j = 1,2), \quad (1)$$

where g is the gravitational acceleration. We analyse linear stability characteristics of the base state described below.

2.1. Base state

The base state corresponds to a steady, parallel, fully-developed flow in both the layers separated by a flat interface, i.e., $V_j = W_j = 0$; U_j is only a function of z and pressure distribution ($P_1 = P_2 = P$) is linear in x.

$$U_1 = \frac{1}{2m}\frac{dP}{dx}\left[z^2 + \left\{\frac{{h^0}^2 + m(1-h^0)}{m(h^0-1) - h^0}\right\}z\right], \quad (2)$$

$$U_2 = \frac{1}{2}\frac{dP}{dx}\left[z^2 - 1 + \left\{\frac{{h^0}^2 + m(1-h^0)}{m(h^0-1) - h^0}\right\}(z-1)\right]. \quad (3)$$

The pressure gradient, dP/dx, is obtained from the constant volumetric flow rate condition, i.e.,

$$\int_0^{h^0} U_1\,dz + \int_{h^0}^{1} U_2\,dz = 1. \quad (4)$$

We obtained Eqs. (2) and (3) by integrating the steady, fully-developed dimensionless Navier–Stokes equations, imposing the no-slip conditions at the walls and demanding continuity of velocity and the tangential component of the stress at the interface. Typical basic state profiles of the steady, streamwise velocity component for $h^0 = 0.3$ are shown in Fig. 2. These parameter values are chosen such that they satisfy $n \equiv h^0/(1-h^0) < \sqrt{m}$ for which there is no Squire's theorem (Yiantsios and Higgins, 1988a). Inspection of

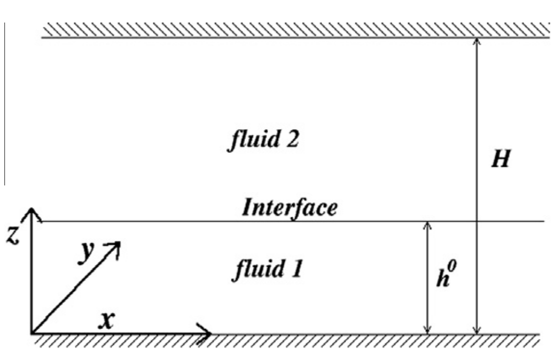

Fig. 1. Schematic of a two-layer flow in a channel of height H, where h^0 represents the thickness of the lower fluid.

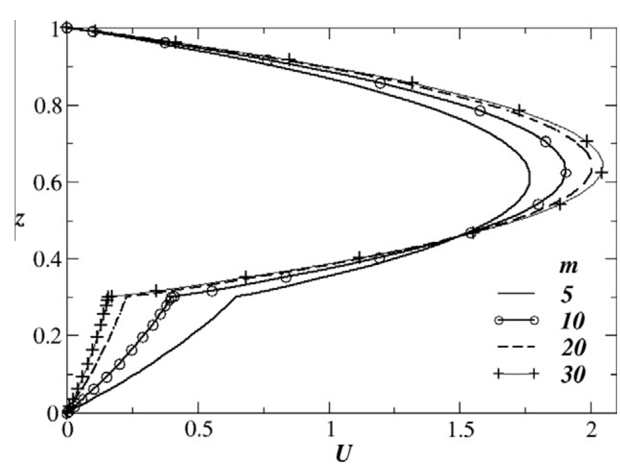

Fig. 2. Basic state profiles of the steady, streamwise velocity profiles for different viscosity ratios. The height of the interface from the bottom wall, $h^0 = 0.3$.