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New model for single spherical particle settling velocity in power law (visco-inelastic) fluids

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Abstract

Particle settling in a non-Newtonian power law fluid is of interest to many industrial applications, including chemical, food, pharmaceutical, and petroleum industry. Conventionally, the Newtonian model for the drag coefficient prediction is extended to non-Newtonian fluids. The approach of merely replacing a viscosity term in Newtonian correlation with a power law apparent viscosity is reported to be inadequate.

In this investigation, the inadequacy of the Newtonian model to correlate the data of single solid spherical particle moving in power law liquids is demonstrated. An approach presented earlier by Shah has been adopted to re-analyze the previously published data of particle settling in various non-Newtonian fluids from five different investigations. The particle settling velocity data have been correlated with two dimensionless quantities – drag coefficient C_d and particle Reynolds number Re – as $\sqrt{C_d^{2-n} Re^2}$ versus Re, rather than the conventional correlation of C_d versus Re. A new model to predict the settling velocity of a spherical particle moving in inelastic power law liquids is presented, which reduces to the expected Newtonian fluid limit. It is shown that the Shah's method predicts the particle settling velocity data much closer to the experimental data than the Newtonian standard drag curve that has been widely used by many researchers. The new model is valid for a wide range of power law flow behavior index n (0.281–1.0) and particle Reynolds number Re (0.001–1000). The paper is concluded by presenting an illustrative example to calculate the settling velocity of a spherical particle in non-Newtonian liquid.

Keywords: Particle settling velocity; Sedimentation; Hydraulic fracturing; Drilling cuttings transport; Power law fluids; Non-Newtonian fluids

1. Introduction

Reliable knowledge of the free settling velocity of spherical particles in fluids is required while performing process design calculations in a range of industrial settings. Typical examples include the design of slurry

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pipelines, of liquid–solid separation equipment, of fluidized bed reactors, falling ball viscometers, etc. Additional examples are found in petroleum engineering applications. During hydraulically fracturing a hydrocarbon-bearing formation, slurry containing solid particles (more commonly known as "proppants") in a non-Newtonian carrier fluid is pumped to keep the created fracture open upon cessation of pumping. The knowledge of particle settling in slurry under static conditions is very essential in the prediction of fracture closure and the final fracture conductivity. The value of the fracture conductivity is very important to the productivity of the fractured well. While adequate information is now available which permits the calculation of the settling velocity of spherical particles in Newtonian fluids (e.g., see Clift et al., 1978; Chhabra, 2006), in some of the afore-mentioned applications (especially in petroleum engineering sector, mine tailing disposal using slurry pipe lines), the liquid exhibits shear-thinning characteristics which is frequently modelled using the usual two-parameter power law fluid model. Admittedly, a sizeable body of knowledge has accrued on the free falling velocity of spherical particles in power law fluids (Chhabra, 1986, 1990, 2006; Dhole et al., 2006), not only the body of information is nowhere near as extensive as that for Newtonian fluids but also it is much less coherent. Consequently, a simple, reliable and widely tested method is not yet available which can be used with confidence over the entire range of conditions. This work sets out to meet this objective. In particular, the purpose of this work is to demonstrate the inadequacy of the generalization of the Newtonian drag curve for the power law fluids using the notion of an apparent viscosity and also to provide a new model for the prediction of spherical particle settling velocity moving in a power law fluid. It is, however, instructive to begin with the terse description of the pertinent studies.

2. Review of previous work

Owing to their wide occurrence in chemical processes and also in petroleum processes, significant work has been reported on the hydrodynamic behavior by numerous authors to understand the mechanism of particles settling in non-Newtonian fluids. The work reported in the chemical industry has been publicized widely while the work reported in the petroleum industry has not been widely known and no attempt has been made to consolidate these results accepted by other industries. The chemical industry literature on particle settling velocity in non-Newtonian fluids prior to 1990 has been critically reviewed and reported by Chhabra (1986, 1990, 2006).

Dallon (1967) presented an empirical correlation relating the drag coefficient of spheres falling at terminal velocity and non-Newtonian Reynolds number. He worked with hydroxyethyl cellulose (HEC), carboxymethyl cellulose (CMC), and polyethylene oxide (PEO) fluids and fitted the rheological data of these fluids with Ellis model fluid. The developed equation has two constants and the terminal velocity calculations require a trial-and-error procedure. Chhabra (1990) refitted Dallon's data using the power law fluid model covering the flow behavior index, n ranging from 0.64 to 0.94 and drag coefficient between 0.46 and 3000.

The empirical correlation between the drag coefficient and particle Reynolds number developed by Prakash (1983) from the data of motion of spheres through various CMC solutions shows an additional dependence on the power law flow behavior index. It was also noted that the data could have been influenced by the confining walls but no wall correction was applied. Peden and Luo (1987) have also reported experimental data of spheres falling in aqueous solutions of CMC and HEC. The two constants in the relationship between drag coefficient and particle Reynolds number were reported to be functions of the power law flow behavior index but the dependence was found to be irregular. Also, their expression does not reduce to the expected limiting behavior for Newtonian fluids. Koziol and Glowacki (1988), Reynolds and Jones (1989) and Machac et al. (1995), have also reported similar results and correlations. But unfortunately none of these correlations have been tested using independent experimental data. More recently, Renaud et al. (2004) have revisited this problem and reported improved predictions for the data reported in chemical engineering literature. Admittedly, their method does yield reasonable predictions, but not only it is complex in form but also requires an iterative solution for the terminal fall velocity.

Yet another more recent study by Kelessidis (2004a) a argues that his proposed equation for predicting the terminal settling velocity of solid spheres in non-Newtonian shear-thinning fluids is in accordance with the drag curve of Newtonian fluids. Kelessidis and Mpandelis (2004) have also proposed a five-parameter implicit model to predict the settling velocity of single particle in pseudoplastic liquids. Their equation is similar to the