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ScienceDirect
International Journal of Multiphase Flow 32 (2006) 1354–1369
International Journal of Multiphase Flow
www.elsevier.com/locate/ijmulflow

Variation of fiber orientation in turbulent flow inside a planar contraction with different shapes

M. Parsheh [a], M.L. Brown [b], C.K. Aidun [c,*]

[a] *Institute of Paper Science and Technology, Georgia Institute of Technology, Atlanta, GA 30332, USA*
[b] *School of Chemical and Biomolecular Engineering, Georgia Institute of Technology, Atlanta, GA 30332, USA*
[c] *G.W. Woodruff School of Mechanical Engineering, Georgia Institute of Technology, Atlanta, GA 30332, USA*

Received 14 October 2005; received in revised form 9 July 2006

Abstract

In this study, we have investigated the influence of shape of planar contractions on the orientation distribution of stiff fibers suspended in turbulent flow. To do this, we have employed a model for the orientational diffusion coefficient based on the data obtained by high-speed imaging of suspension flow at the centerline of a contraction with flat walls. This orientational diffusion coefficient depends only on the contraction ratio and turbulence intensity. Our measurements show that the turbulence intensity decays exponentially independent of the contraction angle. This implies that the turbulence variation in the contraction is independent of the shape, consistent with the results by the rapid distortion theory and the experimental results of axisymmetric contractions. In order to determine the orientation anisotropy, we have solved a Fokker–Planck type equation governing the orientation distribution of fibers in turbulent flow. Although the turbulence variation and the orientational diffusion are independent of the contraction shape, the results show that the variation of the orientation anisotropy is dependent on shape. This can be explained by the variation of the rotational Péclet number, Pe_r, inside the contractions. This quantity is a measure of the importance of the mean rate of the strain relative to the orientational diffusion. We have shown that when $Pe_r < 10$ turbulence can significantly influence the evolution of the orientation anisotropy. Since in contractions with identical inlet conditions the streamwise position where $Pe_r = 10$ depends on the shape, the orientation anisotropy is dependent on the variation of rate of strain in a given contraction. We demonstrate the shape effect by considering contraction with flat walls as well as three contractions with different mean rate of strain variation.

Keywords: Fiber suspension; Orientational diffusion; Fiber orientation; Orientation distribution function; Turbulent flow; Planar contraction; Rotational Péclet number; High-speed imaging

1. Introduction

Motion and orientation of suspended fibers in turbulent flow affect transport, rheology, and turbulence characteristics of the suspension. In addition, in many industrial processes the quality of the final products

* Corresponding author. Tel.: +1 404 894 6645; fax: +1 404 894 4778.
E-mail address: cyrus.aidun@me.gatech.edu (C.K. Aidun).

0301-9322/$ - see front matter
doi:10.1016/j.ijmultiphaseflow.2006.07.002

is dependent on the orientation of fibers. For example, in papermaking, mechanical properties of manufactured paper are known to be anisotropic due to the anisotropy in the orientation distribution of fibers induced by the flow kinematics while passing through a planar contraction. One of the primary parameters which can be changed to alter the orientation distribution function in converging flows is the shape of the contraction.

The dynamics and orientation of an inertialess ellipsoid in a linear shear flow can be obtained from Jeffery's equation (1922). The tensorial form of this equation is given by

$$\dot{p}_i = \Omega_{ij}p_j + \lambda(E_{ij}p_j - E_{kl}p_k p_l p_i), \tag{1}$$

where p_i, Ω_{ij} and E_{ij} denote the unit orientation vector along the longitudinal axis of the particle, the antisymmetric and symmetric part of the velocity gradient tensor, respectively; and $\lambda(\equiv [a_p^2 - 1]/[a_p^2 + 1])$ is a function of the fiber aspect ratio, a_p.

Brenner (1974) generalized this approach to cover any axisymmetric particle. This analysis is valid when the suspension is dilute, i.e., each fiber can rotate freely without affecting the motion of other fibers or its motion become affected by other fibers (Bibbo et al., 1985). This requires $n < 1/l^3$, where n and l denote the number of fibers per unit volume and the fiber half length, respectively. When $n > 1/l^3$, fibers hydrodynamically interact and influence their orientation state. Thus, in a semi-dilute suspension the motion of a fiber depends on the mean fluid velocity, the fluctuating component of the fluid velocity, hydrodynamic fiber–fiber interactions, and inertia.

In order to quantify the effect of hydrodynamic fiber–fiber interactions on orientation, Shaqfeh and Koch (1990) developed a model based on kinetic theory to predict the dispersion of fibers oriented along the extensional axis of axisymmetric and planar extensional flows. This model shows that the orientational dispersion for dilute and semi-dilute suspensions is $O(nl^3/\ln^2 a_p)$ and $O(\ln(nl^3)/nl^3)$, respectively. In the dilute regime, as the concentration increases from infinite-dilute, the rate of dispersion increases. While in the semi-dilute regime the dispersion rate decreases with increasing fiber concentration. This decrease at semi-dilute regime is attributed to the short range screening of hydrodynamic disturbances (see e.g., Shaqfeh, 1988; Shaqfeh and Fredrickson, 1990; Shaqfeh and Koch, 1990). Shaqfeh and Koch also observed that the fiber dispersion in a planar extensional flow is anisotropic in the dilute regime. Dispersion in the transverse direction was shown to be larger than that in the direction of extension.

The orientation state of a large number of fibers is described by the probability distribution function, $\psi(\mathbf{p}, t)$. This function is normalized such that

$$\oint \psi(\mathbf{p}, t)\,\mathrm{d}\mathbf{p} = 1 \tag{2}$$

(Dinh and Armstrong, 1984). Based on the conservation principles in the $\mathbf{p}$ space, distribution function must satisfy the continuity equation given by

$$\frac{\mathrm{D}\psi}{\mathrm{D}t} + \nabla_{\mathrm{r}} \cdot (\dot{\mathbf{p}}\psi) = 0, \tag{3}$$

where ∇_{r} is the gradient operator in orientation space (i.e., the gradient operator of the surface of a unit sphere). Analogous to suspension flows with Brownian motion and with hydrodynamic fiber–fiber interactions, the change of orientation distribution function $\psi(\mathbf{p}, t)$ in turbulent flows can be modeled by a Fokker–Planck type equation (see e.g., Advani and Tucker, 1987; Doi and Edwards, 1988; Krushkal and Gallily, 1988; Koch, 1995; Olson and Kerekes, 1998) given by

$$\frac{\mathrm{D}\psi}{\mathrm{D}t} = \nabla_{\mathrm{r}} \cdot (\mathbf{D_r} \cdot \nabla_r \psi - \dot{\mathbf{p}}\psi), \tag{4}$$

where $\mathbf{D_r}$ is the rotational diffusion coefficient tensor. In this equation, the translational diffusion is neglected because the fiber concentration in the suspension flow is assumed to be homogeneous. Depending on the flow conditions, the diffusion term on the right hand side of (4) represents the randomizing effect due to either the Brownian motion (Doi and Edwards, 1988), the turbulent eddies (Krushkal and Gallily, 1988; Olson and Kerekes, 1998) or the hydrodynamic fiber–fiber interactions (Koch, 1995). In this study, we consider that the rotational diffusion is isotropic and thus is represented by a scalar diffusion coefficient, D_{r}, instead of a tensor.