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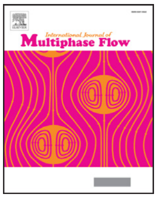

Modeling of the bubbling process in a planar co-flow configuration

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ABSTRACT

This work presents an analytical model developed to describe the bubbling regime resulting from the injection of an air sheet of thickness $2H_o$ with a mean velocity u_a between two water streams of thickness $H_w - H_o$, moving at a uniform velocity u_w. Based on previous experimental and numerical characterizations of this flow, the gas stream is modeled as a two-dimensional sheet divided into three different parts in the streamwise direction: a neck that moves downstream at the water velocity, a gas ligament attached to the injector upstream of the neck, and a forming bubble downstream of the neck, whose uniform dimensionless half-thicknesses are $\eta_n(\tau)$, $\eta_l(\tau)$, $\eta_b(\tau)$ respectively, and the corresponding pressures are given by $\Pi_n(\tau)$, $\Pi_l(\tau)$, and $\Pi_b(\tau) = \Pi_n(\tau)$. Lengths are made dimensionless with H_o, and pressures with $\rho_a u_a^2$, where ρ_a is the air density. In a reference frame moving with the water velocity, and imposing a negative pressure caused by the sudden expansion of the air stream at the outlet of the injector, a set of algebraic-differential equations are deduced, that can be numerically integrated to obtain the temporal evolution of the interface positions and gas pressures, as well as of the gas flow rate through the neck. The model shows a good agreement with previous experimental and numerical results for a given value of the initial velocity of the collapsing neck, determined by an iterative method that matches the bubbling time with that given by Gutiérrez-Montes et al. (2013), $\tau_b^c = 9.1\,\Lambda\sqrt{(\rho_w/\rho_a)(h-1)/[\mathrm{We}\,(\beta-\beta^2)]}$. Here $\Lambda = u_w/u_a$ is the water-to-air velocity ratio, $\mathrm{We} = \rho_w u_w^2 H_o/\sigma$ the Weber number, $h = H_w/H_o$ the water-to-air thickness ratio and $(1-\beta) = (H_o - H_i)/H_o$ the dimensionless wall thickness of the air injector.

Introduction

Generation of gas bubbles in a liquid is one of the most important and common operations in many industrial applications, such as aeration, distillation, or absorption, traditionally used in material, mineral, chemical or food industries, among many others. In addition, in the last few years, a number of emerging technologies related with the medical and the pharmaceutical industries demand the generation of small monodisperse bubbles, justifying the need of a deeper understanding of the bubble size control (Rodríguez-Rodrıguez et al., 2015). The simplest and most studied method to generate bubbles consists of introducing the gas stream through an injector which discharges inside a still liquid medium (see Davidson and Schuler, 1960; Kumar and Kuloor, 1976; Longuet-Higgins et al., 1991; Oguz and Prosperetti, 1993; Kulkarni and Joshi, 2005; Bolaños-Jiménez et al., 2008, among others). However, this method only allows the controlled production of bubbles at frequencies much smaller, and bubble sizes much larger, than those required by most of the modern applications mentioned above.

One of the most extended methods to generate smaller and monodisperse bubbles is the well-known *co-flow* technique, where the gas discharges inside a laminar stream of liquid which flows in the same direction. This configuration allows to inject higher gas flow-rates compared to the case of still liquid, while avoiding bubble coalescence and irregular bubbling regimes. The classical co-flow configuration with a cylindrical geometry has been extensively studied and it is used in many applications (Maier, 1927; Chuang and Goldschmidt, 1970; Oguz and Prosperetti, 1993; Gordillo et al., 2001; Sevilla et al., 2002; 2005a; 2005b), including microfluidic devices (Stone et al., 2004; Gordillo et al., 2001; 2004). Nevertheless, a planar co-flow configuration, which is the case studied in the present work, represents an alternative method to produce controlled-size bubbles (Bolaños-Jiménez et al., 2011; Gutiérrez-Montes et al., 2013; 2014). In this configuration, which has been comparatively less studied than the cylindrical one, a planar air film discharges between two parallel water sheets. As in the

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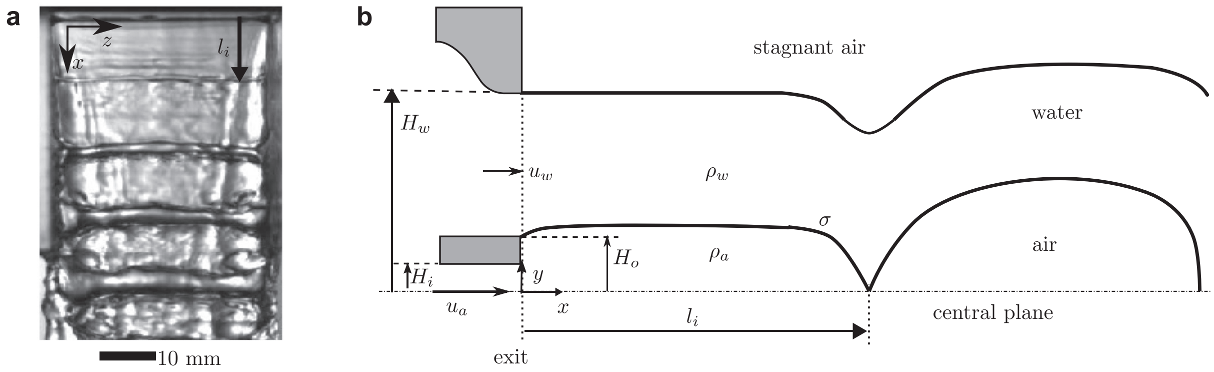

Fig. 1. (a) Experimental image of the bubbling regime in the planar co-flow configuration. (b) Sketch of the planar bubbling process with the main geometrical and physical parameters.

cylindrical case, Bolaños-Jiménez et al. (2011) observed the existence of two different flow regimes: a *jetting regime*, where the air sheet does not break near the injector, and a *bubbling regime*, where a periodic and quasi-two-dimensional break-up of the air sheet into individual bubbles is observed (see Fig. 1(a)). In addition, they characterized the jetting-to-bubbling transition in the $\mathrm{We}-\Lambda$ parameter space where $\mathrm{We}=\rho_w u_w^2 H_o/\sigma$ is the Weber number and $\Lambda=u_w/u_a$ the liquid-to-gas mean velocity ratio. Here, ρ_w is the water density, σ the surface tension coefficient, and H_o the half-thickness of the air stream at the exit slit (see Fig. 1(b)). Unlike in the cylindrical configuration where surface tension effects contribute to destabilizing the air–water jet (Sevilla et al., 2005b), they stabilize the water–air–water sheet in the planar case. Moreover, a local linear stability analysis revealed that the flow transition is related to the convective or absolute nature of the local instability in the near field.

The dynamics of the bubbling regime in the planar configuration was also investigated by Gutiérrez-Montes et al. (2013) by means of experiments and numerical simulations for a particular case with prescribed values of the dimensionless geometrical parameters, $h=H_w/H_o$ and $\beta=H_i/H_o$, where H_w is the distance of the water interface to the central plane, H_i and H_o the inner and outer semi thicknesses of the air injector respectively being, thus, H_o-H_i the wall thickness of the air injector (see Fig. 1(b)). Based on the temporal evolution of the bubble shape and the gas pressure extracted from the numerical simulations performed by Gutiérrez-Montes et al. (2013), the bubble formation event was described as a two-stage process: the *neck formation* and the subsequent *neck collapse* stages. The former starts just after the pinch-off of the previous bubble, when an initial air lump of length l_i, called *intact ligament*, remains attached to the outer wall of the air nozzle (see Fig. 1). Therefore, the gas stream suffers a sudden expansion from the inner thickness of the air injector, $2H_i$, to the outer one, $2H_o$, inducing a persistent negative gauge pressure inside the air stream in the neighborhood of the injector exit. As a consequence, an incipient neck appears, that propagates downstream at the water velocity while it accelerates inwards, causing a pressure drop across it. Thus, in order to keep the feeding air flow rate constant, the gas pressure at the exit has to increase. This process continues in time and during the *collapse stage* it becomes more violent, inducing the inflation of the air ligament upstream from the neck and, consequently, decreasing the air flow rate that passes through the neck.

Based on the above description, Gutiérrez-Montes et al. (2013) proposed a scaling law for the characteristic bubbling time, given by $t_c \propto H_o/u_a\sqrt{(\rho_w/\rho_a)(h-1)/[\mathrm{We}\,\beta(1-\beta)]}$, which was shown to reproduce fairly well the experimental and numerical bubbling times. In this scaling law the pressure loss associated with the planar sudden expansion is the only mechanism taken into account to cause the pressure decrease at the injector tip. However, in the cases where the relative wall thickness is very small, $1-\beta\ll 1$, when the effect of the sudden expansion is not dominant, alternative phenomena leading to negative gauge pressures in the air stream determine the bubbling time, such as the Bernoulli suction through the neck (Venturi effect) or the elongation of the growing bubble, as already pointed out in Gutiérrez-Montes et al. (2013, 2014). Regarding the Bernoulli suction, although it is dominant during the last instants of the bubble collapse, it can not account for the neck formation stage, as happens in the cylindrical configuration (Sevilla et al., 2005a; Gordillo et al., 2005; 2007). Concerning the elongation of the forming bubble, in Gutiérrez-Montes et al. (2013) it was already elucidated that a negative pressure is only possible when the length of the forming bubble increases with time, in contrast with the cylindrical case (Gordillo et al., 2007). The relative importance of the different suction mechanisms mentioned above depends on the specific geometry of the bubble, cylindrical or planar, and on details of the injection system, such as the thickness of the walls separating the air and water streams at the nozzle exit, as discussed in Section 3.4 in (Gutiérrez-Montes et al., 2013).

The main goal of the present work is to extend the theoretical understanding of the planar bubbling regime. To that end, we propose a simple analytical model incorporating the main physical mechanisms that determine the bubbling process at constant gas flow rate. In contrast with most of the previous efforts to model the bubbling phenomenon, which are based on global force balances (Davidson and Schuler, 1960; Ramakrishnan et al., 1968; Chuang and Goldschmidt, 1970; Kumar and Kuloor, 1976; Marmur and Rubin, 1970; Terasaka and Tsuge, 1993; Tan and Harris, 1986), our approach is similar to that developed in Gordillo et al. (2007) for the cylindrical case.

The work is organized as follows. The analytical model is described in detail in Section 2, while an evaluation of the model, including comparisons with experimental and numerical results, is shown in Section 3. Finally, Section 4 summarizes the main conclusions.

Model description

To model the bubbling process, and based on the information extracted from previous experiments and numerical simulations, the simplified flow configuration sketched in Fig. 2 will be considered. Bolaños-Jiménez et al. (2011) found that, in the planar configuration, surface tension played a relevant role in the transition from a jetting to a bubbling regime. In fact, they observed that, as the Weber number increased, i.e. the relative influence of surface tension decreased, the shear required to achieve a jetting regime was smaller, manifesting the stabilizing surface