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Overview of existing Langevin models formalism for heavy particle dispersion in a turbulent channel flow

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ABSTRACT

The purpose of the paper is to compare two successful families of stochastic model for the prediction of inertial particles dispersion in a turbulent channel flow. Both models are based on the Langevin equation; nevertheless, they were developed following different paths. The first model considered is named "Drift Correction model (DCM)", and the second one is the "Generalized Langevin Model (GLM)". To examine the capabilities of both models, a comparison of the results predicted by the DCM- and GLM-type dispersion models with those extracted from a Direct Numerical Simulation (DNS) is conducted. In the limit of vanishing particle inertia, both models can accurately predict second-order statistics. It is also noticed, as not expected, that they are very similar when they are written in the same functional form. The comparison has also been conducted with DNS data of a particle-laden channel flow. The comparison of particle statistics (such as concentration, mean and rms particle velocity, third-order particle velocity correlations) shows that both stochastic models give very satisfactory results up to second-order statistics. The DCM- and GLM-type dispersion models studied can capture the main physical mechanisms that govern particle-laden turbulent channel flows.

1. Introduction

From physical observation of Brownian motion (Brown, 1828) up to the first mathematical object (Kiyoshi Itô), 146 years have been necessary to obtain a mathematical object of practical use, i.e. a Markov diffusion stochastic process, $\mathbf{X}(t)$, which verifies the following stochastic differential equation (SDE):

$$\mathrm{d}X_i(t) = A_i(\mathbf{X}(t), t)\mathrm{d}t + B_{ij}(\mathbf{X}(t), t)\mathrm{d}W_j(t), \tag{1}$$

where A_i and B_{ij} are the drift and diffusion coefficients, respectively. The first term on the right-hand side is called "drift term", and the second is the "diffusion term" which is a rapidly varying component. $\mathrm{d}W_j$ are the increments of a vector-valued Wiener process with independent components. These increments are non-differentiable and normally distributed with zero mean and covariance $\langle \mathrm{d}W_i \mathrm{d}W_j\rangle = \mathrm{d}t\,\delta_{ij}$. This SDE is a powerful tool in science, mathematics, economics and finance and is used for modeling various processes in the physical world. Examples are stock market evolution, molecular dynamics or turbulence modeling for unladen and laden flows.

To introduce the use of this approach for dispersed two-phase flows, we first consider the description of the Brownian motion of a particle in a fluid at rest using a diffusion process such as Eq. (1). The one-dimensional particle motion can be described by the following set of equations:

$$\begin{cases} \mathrm{d}x_p = \upsilon_p \mathrm{d}t, \\ \mathrm{d}\upsilon_p = -\dfrac{\upsilon_p}{\tau}\mathrm{d}t + \left(\dfrac{2\langle {\upsilon_p}^2\rangle}{\tau}\right)^{\frac{1}{2}} \mathrm{d}W, \end{cases} \tag{2}$$

where $x_p(t)$ and $\upsilon_p(t)$ are the particle location and velocity, the time τ is the local decorrelation timescale of $\upsilon_p(t)$. In the functional form of the SDE used to model the time increment of the particle velocity, a linear function of particle velocity for the drift term $A = -\upsilon_p(t)\tau^{-1}$ is stated and the diffusion term is defined as a function of the particle velocity variance as $B^2 = 2\langle {\upsilon_p}^2\rangle\tau^{-1}$. This type of diffusion process (Eq. (2)) has progressively been used over the years to develop simple model of the motion of fluid elements in homogeneous isotropic turbulence, and more complicated ones that take turbulence anisotropy and inhomogeneity into account.

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The same functional form as Eq. (2) is applied; however, the parameters (drift and diffusion) have to be re-interpreted. For instance, in homogeneous and stationnary turbulence with a zero mean fluid velocity, the local decorrelation time scale τ is identified to be the velocity Lagrangian time scale T_L. Therefore, the Langevin-type equation used to model the instantaneous fluid velocity (u_i) increments for such a turbulent flow can be written as (Durbin, 1980a, 1980b; Sawford, 1984, 1985):

$$\mathrm{d}u_i = -\frac{u_i}{\tau}\mathrm{d}t + \left(\frac{2\langle u_i'^2\rangle}{\tau}\right)^{\frac{1}{2}}\mathrm{d}W, \tag{3}$$

where $\tau = T_L$, and u_i' is the fluctuating part of the fluid velocity. The input parameters for the above Langevin model are thus the variance ($\langle u_i'^2\rangle$) of the velocity and the Lagrangian integral timescale. Since the 1980s, it has been shown that the direct use of Eq. (3) to predict the diffusion of fluid particles in non-homogenous turbulent flows is erroneous. Such a model does not give realistic results since particle concentration tends to increase in regions with low velocity variance in violation with the second law of thermodynamics (spurious drift effect, cf. Pope, 1987). To fill this gap, two kinds of Langevin equations which are both based on Eq. (3) were proposed. Wilson et al. (1981) and Legg and Raupach (1982) introduced directly a corrective term in the stochastic equation in order to avoid the spurious drift effect. As noted by Rodean (1994), both models used physical reasoning, but the corrective terms are ad hoc addition to the Langevin equation. We name this group of models "Drift correction models (DCM)". We gather the models by Wilson et al. (1981) and Legg and Raupach (1982) in the same group but it has to be kept in mind that they are based on different formulations of the Langevin equation. A normalized formulation of the Langevin equation was proposed by Wilson et al. (1981) in opposite to that done by Legg and Raupach (1982). For instance, reduced to the wall-normal direction, y, in a boundary layer (where the mean wall-normal velocity is supposed to be equal to zero), the normalized formulation proposed by Wilson et al. (1981) is:

$$\mathrm{d}\left(\frac{v}{\sigma_v}\right) = -\frac{v}{\sigma_v}\frac{dt}{\tau} + \left(\frac{2}{\tau}\right)^{\frac{1}{2}}\mathrm{d}W + \frac{d\sigma_v}{dy}\mathrm{d}t, \tag{4}$$

where $\sigma_v{}^2 = \langle v'^2\rangle$ and the last term of this equation is the correction term. From simple mathematical manipulations, Eq. (4) can be written as:

$$\mathrm{d}v = -\frac{v}{\tau}\mathrm{d}t + \sigma_v\left(\frac{2}{\tau}\right)^{\frac{1}{2}}\mathrm{d}W + \sigma_v\left[\left(\frac{v}{\sigma_v}\right)^2 + 1\right]\frac{d\sigma_v}{dy}\mathrm{d}t. \tag{5}$$

Such a functional formulation can be compared to that proposed by Legg and Raupach (1982):

$$\mathrm{d}v = -\frac{v}{\tau}\mathrm{d}t + \sigma_v\left(\frac{2}{\tau}\right)^{\frac{1}{2}}\mathrm{d}W + \frac{d\sigma_v^2}{dy}\mathrm{d}t. \tag{6}$$

We observed that for a weakly turbulent flow $v\sigma_v^{-1} \to 1$, Eq. (5) tends toward Eq. (6). Although the functional form of these two terms are different, they have the same physical meaning. They represent a mean force due to the action of the mean pressure gradient on fluid particles. To summarize the scientific road map before 1983, the extension to non-homogeneous turbulence was carried out by taking the space dependence of parameters τ and $\langle u_i'^2\rangle$ into consideration, and by correcting the model to avoid unphysical effects.

There has been a real advance in the stochastic modeling of turbulent fluid dispersion for inhomogeneous flows from 1983. We owe it to the seminal works of Pope (1983, 1985, 1987) and Thomson (1984, 1987) who have raised the issue of the consistency that may exist between the velocity probability density function (PDF) of tracer particles (noted P), tracked through the use of a Langevin-type model, and the velocity probability density function of the flow (noted P_f). Thomson (1987) puts forward the famous well-mixed condition (WMC) that must be met by a Langevin equation to model the dispersion of tracer particles in a turbulent flow in order to ensure the consistency with the flow. This condition states that trace material initially well mixed in a fluid must remains so. In other words, it means that the joint probability distribution of position and velocity of tracer particles will remain the same as that of the fluid particles. Under this condition, fluid particles and tracers must have the same velocity moments, position moments, and joint moments, which characterizes the consistency. In other terms, it strictly means that the probability density function of tracer particles, P, is equal to the probability density function P_f of the fluid particles. The Fokker-Planck equation is then satisfied by P (noted g_a in his paper), and the drift parameter of the Langevin equation can be determined to satisfy the WMC. However, the main drawback is that this is possible only if the form of the velocity PDF is known beforehand.

Pope (1987) is rather critical about this well-mixed condition. The same year as the famous paper of Thomson, Pope publishes an article "Consistency conditions for random-walk models of turbulent flows" (Pope, 1987) which is clearly a response to the article of Thomson : "*My view was (and is) that a minimal requirement of a stochastic model is that it be consistent with the mean momentum equation. How to make models consistent was well known in the PDF literature in the 1980s, i.e., by incorporating mean pressure and Reynolds stress gradients correctly. A model that is thus consistent automatically satisfies the "well mixed condition"*."(Pope, Personal communication, 2009).

Pope proposed the following functional form of the Langevin-type model for the instantaneous fluid velocity increment,

$$\mathrm{d}u_i = \left[-\frac{1}{\rho}\frac{\partial\langle p\rangle}{\partial x_i} + \nu\nabla^2\langle u_i\rangle\right]\mathrm{d}t + A_i\mathrm{d}t + B_{ij}\mathrm{d}W_j, \tag{7}$$

where $p(x, t)$ is the pressure, ρ is fluid density, and ν represents the fluid kinematic viscosity. The drift term is linear and equal to $A_i = -G_{ij}[u_j - \langle u_j\rangle]$, where G_{ij} has an inverse time dimension. It can be noted that the mean pressure gradient appears naturally in the stochastic model. In Pope's sense, WMC is satisfied as soon as the calculated mean velocity from the Langevin equation satisfies the continuity equation. This, in turn, requires a correct introduction of the mean pressure gradient in the Langevin equation. Pope (1987, 2002) also provided an important algebraic relation between the drift and diffusion terms which ensures the consistency between the Langevin-type model and second order Eulerian statistics of the turbulent flow. Pope gives the following relation for any turbulence:

$$G_{jk}\langle u_i'u_k'\rangle + G_{ik}\langle u_j'u_k'\rangle + B_{ik}B_{jk} = +\nu\frac{\partial^2}{\partial x_k\partial x_k}\langle u_i'u_j'\rangle - \frac{1}{\rho}\left[\left\langle u_i'\frac{\partial p'}{\partial x_j}\right\rangle + \left\langle u_j'\frac{\partial p'}{\partial x_i}\right\rangle\right] - 2\nu\left\langle\frac{\partial u_i'}{\partial x_k}\frac{\partial u_j'}{\partial x_k}\right\rangle. \tag{8}$$

From this last relation, a compatibility between the Langevin-type model and second-moment closure models can be provided. This group of models is named Generalized Langevin Models, noted *GLM* where the drift and diffusion parameters have to be specified. Note that no assumption is made about the form of the velocity distribution in opposite to Thomson's approach.

Thanks to these pioneering works, the connection between Langevin equation and turbulent flow has been made for fluid particle diffusion in non-homogeneous turbulent flows. From a literature review on inertial particle turbulent dispersion, it is clear that the models for the time increment of the fluid velocity along solid particle trajectory were derived from the models