J. of Supercritical Fluids xxx (2015) xxx–xxx

Contents lists available at ScienceDirect

The Journal of Supercritical Fluids

journal homepage: www.elsevier.com/locate/supflu

Mixed convection on a vertical plate in supercritical fluids by selecting the best equation of state

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ARTICLE INFO

Article history:
Received 29 May 2015
Received in revised form 12 July 2015
Accepted 13 July 2015
Available online xxx

Keywords:
Supercritical fluids
Mixed convection
Nusselt number
Upward and downward flow
Richardson number

ABSTRACT

The present article investigates laminar mixed convection with upward and downward flow over a vertical flat plate in supercritical fluid, numerically. At first four different equations for thermal expansion coefficient estimation in supercritical fluids are derived as a function of pressure, temperature and the compressibility factor. Calculated values of thermal expansion coefficient and Nusselt number have been compared with the experimental results and show better accuracy of Redlich–Kwong EOS in comparison with others. After that governing equations for mixed convection are solved numerically by using finite difference method. A parametric study is performed to illustrate the interactive influence of the governing parameters, specifically the Richardson number and the effect of its variation on the rate of heat transfer and Nusselt number in supercritical condition. Finally, heat transfer curves as a function of Richardson number are plotted and observed that general trend of supercritical fluids curves are similar for water and carbon dioxide. The achieved results can also be validated with shown velocity and temperature counters and vectors.

1. Introduction

Convection along a heated vertical flat plate is one of the most common heat transfer phenomena in heat and mass transfer studies, due to its occurrence in a wide variety of heat transfer industrial equipment. If the natural convection existing in this case is accompanied by a forced flow, the combined mode of free and forced convection will exit, which is usually called as mixed convection. Depending on the forced flow direction, the buoyancy forces may aid or oppose the forced flow, causing an increase or decrease in heat transfer rates [1]. So, all mechanisms that enhance natural or forced convection can enhance mixed convection and vice versa. Using fins and ribs to increase the surface area, ultrasonic vibrating of working fluid, applying electrical field, adding surfactants or nanoparticles to the base fluid and finally, using base fluid in supercritical conditions are the most common methods, which have been used to improve one of the free and forced convection or both of them and finally enhance mixed heat transfer mechanism [2–7].

The theory of mixed convection shows that the ratio of the Grashof number to the square of Reynolds number which is known as the non-dimensional Richardson number, has a great effect on the flow with the constant plate's temperature boundary condition. The forced convection is dominating in small values of the Richardson number because of large values of Reynolds number; on the other hand, free convection overcomes in large values of the Richardson number because of large values of Grashof number.

Reviewing the literatures shows that the mixed convection theory from a flat plate for under critical condition is well established and has been investigated by various researchers. Sparrow et al. [8] were the first to study the effect of the buoyancy forces on the forced convection flow over a vertical flat plate. The results of their study have shown that the flow can be classified by using the Richardson number. A combination of numerical integration and series expansion has been used by Merkin [9] and Wilks [10] to solve the mixed convection problem for both aiding and opposing flows in a Prandtl number of unity along a vertical plate with constant wall temperature and constant heat flux boundary condition, respectively. Lioyd and Sparrow [11] used the local similarity method for different values of the Prandtl number to discuss the mixed convection under small effect of Richardson number. Oosthuizen and Hart [12] solved the constant wall temperature and the heat flux problem over a vertical flat plate, numerically. First experimental results of the mixed convection along a vertical plate for air have been presented by Gryzagoridis [13] in a wide range of Richardson number. The comprehensive review concerning mixed convective

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http://dx.doi.org/10.1016/j.supflu.2015.07.013
0896-8446/

Nomenclature

a,b	EOS constants that corrects for intermolecular attractive forces (N m^4/mol^2)
A,B	dimensionless EOS parameters
C_p	heat capacity of the fluid (J/kg K)
C_p^0	heat capacity at the low-pressure limit (J/kg K)
g	local acceleration of gravity (m/s^2)
Gr	Grashof number
h	heat transfer coefficient (W/m^2 K)
k	thermal conductivity of the fluid (W/m K)
k^0	thermal conductivity at the low-pressure limit (W/m K)
L	height of the vertical plate (m)
M	molecular weight of the fluid (kg/mol)
Nu	Nusselt number
P	pressure (Pa)
Pr	Prandtl number
R	gas constant (J/mol K)
Ra	Rayleigh number for the whole plate
Ri	Richardson number
Re	Reynolds number
T	temperature (K)
ρ	density of the fluid (kg/m^3)
ω	acentric factor
u,v	velocity components in the *x,y* direction, respectively (m/s)
V'	molar volume (m^3/mol)
u',w	EOS parameter
U,V	dimensionless velocity components in the *x* and *y* directions respectively
x,y	coordinates parallel and normal to the vertical plate, respectively (m)
X,Y	dimensionless coordinate parallel and normal to the vertical plate, respectively
Z	compressibility factor

Greek symbols

α	thermal diffusivity (m^2/s)
β	thermal expansion coefficient (1/K)
ν	kinematic viscosity (m^2/s)
Γ	inverse thermal conductivity (m K/W)
θ	dimensionless temperature
ξ	inverse viscosity (m^2/N s)

Subscripts

c	critical condition
f	fluid
i	node index in the *x* direction
j	node index in the *y* direction
P	constant pressure
r	reduced characteristic
w	wall
x	local distance
∞	outside the boundary layer

flow for under critical fluids are available in the books by Pop and Ingham [14]. Jena and Mathur [15] solved laminar mixed convection problem of a micropolar fluid from an isothermal vertical flat plate by using a finite-difference technique. They found that the velocity increased and the temperature decreased with the process of fluid's heating.

Many investigations have been done on the effectiveness of supercritical fluids in mixed convection heat transfer; however, there are no reports about the mixed convection of supercritical fluid over a vertical flat plate. Kakaral and Thomas studied mixed convection heat transfer in a vertical tube of supercritical fluids. They evaluated the effect of flow direction at rate of heat transfer in tube [16]. Pioro and Duffey's work [17] is considered as a very useful review literature of supercritical heat transfer phenomena until now. The mixed convective heat transfer of carbon dioxide at supercritical state in a circular vertical and horizontal tube was investigated experimentally and numerically, respectively [18,19]. These studies concerned the effects of the Richardson number on the convective heat transfer in tubes. They both used this fact that the thermophysical properties of supercritical fluids are highly sensitive to temperature.

To study the mixed convection in under critical fluids with temperature-dependent properties over a vertical flat plate; at first, Kafoussias et al. [20] used a modified numerical solution to study the problem with temperature-dependent viscosity. Pantokratoras [21] assumed the temperature-dependent density to study the laminar mixed convection problem of water along a vertical isothermal plate. Mixed convection laminar boundary layer flow of water-based nanofluids along the vertical plate with temperature-dependent heat source has been investigated numerically by Rana et al. [22].

As mentioned before, in supercritical condition the properties of the fluid specially the thermal expansion coefficient are highly related to temperature and pressure. The thermal expansion coefficient (β) has been assumed to be constant and its variation versus temperature and pressure has been neglected in most of previous works. In supercritical fluids, this consideration will lead to wrong results. So, it is evident that the determination of the value of β through an appropriate equation of state is necessary. The effect of temperature, pressure and the compressibility factor (Z) on thermal expansion coefficient was first considered by Rolando in the problem of free convection over an isothermal vertical plate [23]. In his research a new equation for the thermal expansion coefficient has been derived based on the Van der Waals equation of state. Teymourtash and Ebrahimi Warkiani [24] have investigated the effect of linear variation of wall temperature on the heat transfer by natural convection along a vertical plate in supercritical fluids. Teymourtash et al. [25] have carried out the same problem with uniform and non-uniform surface heat flux. For the first time they proved that the Boussinesq approximation is applicable over a wide range of temperature and pressure variation for supercritical fluid. They showed that the Redlich–Kwong equation of state can estimate the supercritical fluid properties much better than Van der Waals or real gas ones. They also found that linearly increasing and decreasing heat flux will increase and decrease the local Nusselt number, respectively, in comparison with the constant ones.

Among all equations of state (EOS) which used to estimate the fluid's properties only real gas, Van der Waals and Redlich–Kwong ones have been used in previous researches for supercritical fluids [23–25]; however, there are some other useful equation of states that can be used in heat transfer estimation at supercritical condition. So, first exclusive objective of this literature is to find the best equation of state among Virial (1901), Beattie–Bridgman (1928), Redlich–Kwong (1949), Soave (1972) and Peng–Robinson (1976) [26] equations for predicting the supercritical fluid's thermal expansion coefficient. After that, the problem of mixed convection along a vertical flat plate is solved for the first time by using a numerical model based on a finite-difference formulation and the most appropriate equation of state. Finally, numerical results are presented and effects of Richardson number on heat transfer characteristics in aiding and opposing mixed flow are investigated.