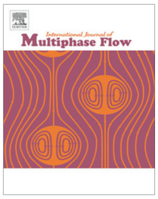
International Journal of Multiphase Flow 76 (2015) 47–63

Contents lists available at ScienceDirect

International Journal of Multiphase Flow

journal homepage: www.elsevier.com/locate/ijmulflow



Displacement of yield-stress fluids in a fracture

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A R T I C L E   I N F O

Article history:
Received 26 May 2014
Received in revised form 30 June 2015
Accepted 5 July 2015
Available online 8 July 2015

Keywords:
Hele-Shaw cell
Fingering
Interfacial instability
Saffman–Taylor instability
Viscous fingering
Yield stress

A B S T R A C T

We consider a displacement of several yield-stress fluids in a Hele-Shaw cell. The topic is relevant to the development of a model for the flow of multiple phases inside a narrow fracture with application to hydraulically fracturing a hydrocarbon-bearing underground formation. Existing models for fracturing flows include only pure power-law models without yield stress, and the present work is aimed at filling this gap. The fluids are assumed to be immiscible and incompressible. We consider fluid advection in a plane channel in the presence of density gradients. Gravity is taken into account, so that there can be slumping and gravitational convection. We use the lubrication approximation so that governing equations are reduced to a 2D width-averaged system formed by the quasi-linear elliptic equation for pressure and transport equations for volume concentrations of fluids. The numerical solution is obtained using a finite-difference method. The pressure equation is solved using an iterative algorithm and the Multigrid method, while the transport equations are solved using a second-order TVD flux-limiting scheme with the superbee limiter. This numerical model is validated against three different sets of experiments: (i) gravitational slumping of fluids in a closed Hele-Shaw cell, (ii) viscous fingering of fluids with a high viscosity contrast due to the Saffman–Taylor (S–T) instability in a Hele-Shaw cell at microgravity conditions, (iii) displacement of Bingham fluids in a Hele-Shaw cell with the development of fingers due to the S–T instability. Good agreement is observed between simulations and laboratory data. The model is then used to investigate the joint effect of fingering and slumping. Numerical simulations show that the slumping rate of yield-stress fluid is significantly less pronounced than that of a Newtonian fluid with the same density and viscosity. If a low-viscosity Newtonian fluid is injected after a yield-stress one, the S–T instability at the interface leads to the development of fingers. As a result, fingers penetrating into a fluid with a finite yield stress locally decrease the pressure gradient and unyielded zones develop as a consequence.

Introduction

Multiphase flows of yield-stress fluids are widely encountered in nature and industry. For example, during extrusive volcano eruptions, cooling magma flowing down a volcano slope demonstrates a highly non-Newtonian yield-stress behavior (Balmforth et al., 2006a,b). Multiphase flows of yield-stress suspensions are also typical for a variety of emerging technologies in the petroleum industry. Hydraulic fracturing of oil or gas reservoirs require modeling suspension flows in naturally-fractured reservoirs (Biryukov and Kuchuk, 2012) and hydraulic fractures (Adachi et al., 2007). Cementing and drilling applications require modeling flows in a narrow annular gap (Bittleston and Hassager, 1992; Pelipenko and Frigaard, 2004a,b; Carrasco-Teja et al., 2008; Carrasco-Teja and Frigaard, 2010; Malekmohammadi et al., 2010).

In this study, we consider a flow of yield-stress fluids in a hydraulic fracture with permeable walls, with application to hydraulically fracturing a hydrocarbon-bearing underground formation. As we focus on the fluid mechanics problem, a hydraulic fracture is approximated as a Hele-Shaw cell with plane walls and a width constant in time but variable in space. Theoretical analysis of such flows requires the formulation of mathematical models accounting for several physical phenomena, which include Bingham fluid flow in a narrow channel, fluid–fluid displacement with the development of instability at the interface, gravitational slumping of a heavier fluid in a lighter one, particle transport and settling in a suspension flow through a fracture, dehydration and granular packing in suspensions due to fluid leak-off into surrounding rock formation. Each of these phenomena is well studied, but to the best of our knowledge they have never been considered in this particular combination. In hydraulic fracturing applications, there are existing models, which account for non-Newtonian rheology of fracturing fluids, but only in the power-law formulation

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http://dx.doi.org/10.1016/j.ijmultiphaseflow.2015.07.001
0301-9322/

(Adachi et al., 2007). Although a typical fracturing fluid is usually a cross-linked polymer water-based solution behaving in rheometry experiments as a Herschel–Bulkley fluid (Economides and Nolte, 2000), the yield stress is not taken into account when modeling fracture flow and a simplified power-law model is typically used (Adachi et al., 2007). Furthermore, fluid–solid suspensions typically exhibit a yield stress at low shear rates, whereas at high shear rates the presence of solids typically causes shear-thinning behavior. The recent development of hydraulic fracturing technology requires one to use more sophisticated chemical compositions for fracturing fluids, which increasingly point out the importance of the yield-stress behavior.

A 2D width-averaged model for suspension flow in a fracture is typically constructed for hydraulic fracturing simulators as follows. In the regime of non-inertial settling, the momentum conservation equation for particles is reduced to a steady-state drift-flux relation for particle velocity, expressed via the fluid velocity and the particle settling velocity. The latter is given by the Stokes formula with a correction for hindered-settling effects at a finite particle volume fraction. In the case of Newtonian suspension rheology, the total momentum conservation equation for the suspension is reduced to the linear expression of the fluid (or mixture) velocity through the pressure gradient, which is similar to Darcy's law, as there is a well-known analogy between the averaged 2D equations for fluid flow in a narrow gap and the 2D model for flows in a porous medium (Saffman and Taylor, 1958). The resulting 2D system of governing equations for particle-laden flow in a narrow plane channel involves the advection equation for particle volume fraction and the elliptic equation for pressure (Pearson, 1994; Hammond, 1995; Boronin and Osiptsov, 2010). The dependence of the width-averaged fluid velocity on the pressure gradient is nonlinear if the rheology of the carrier fluid or of the suspension as a whole is non-Newtonian (see, e.g., Frigaard et al., 2010; Frigaard, 1998; Allouche et al., 2000; Gorodtsov and Yentov, 1997). A review of analytical solutions for the velocity profile of particle-free Bingham fluid flows in simple geometries is given in (Bird et al., 1983).

The flows of particle-free yield-stress fluids in a narrow-gap geometry have been extensively studied with application to fluid displacement in an annulus for cementing of oil and gas wells, both theoretically (Bittleston and Hassager, 1992; Allouche et al., 2000; Bittleston et al., 2002; Carrasco-Teja et al., 2008; Carrasco-Teja and Frigaard, 2010) and experimentally (Malekmohammadi et al., 2010). In (Carrasco-Teja et al., 2008), the displacement of two Herschel–Bulkley fluids in a cylindrical eccentric annulus (between two tubes) is considered in the presence of small inclination of the tubes (i.e. close to horizontal). Issues with the diffusion of fluids at the front obtained in the numerical simulations are discussed, and the effect of gravitational slumping is analyzed. In (Carrasco-Teja and Frigaard, 2010), the model (Carrasco-Teja et al., 2008) is extended to take into account the motion of the inner cylinder. To a lesser extent, this problem has been considered in a plane channel geometry (Amadei and Savage, 2001; Frigaard, 1998), which is typically used to approximate a hydraulic fracture. In particular, a 1D analytical solution was obtained in (Amadei and Savage, 2001) for Bingham fluid flow in a fracture. Buoyancy driven flows of two Bingham fluids in an inclined plane channel were considered in (Frigaard, 1998), with application to the plug cementing process. Generally, when numerically solving the equations of a visco-plastic flow, there is a known problem of regularization, which was considered in detail in (Frigaard and Nouar, 2005). Lubrication theory for a flow in a 2D channel of slowly varying width was developed in (Frigaard and Ryan, 2004). The history of the lubrication paradox (existence or non-existence of a truly unyielded plug region) was discussed there in detail, and an asymptotic solution for the Poiseuille flow was obtained under the assumption that the perturbation of the channel width is small. In application to fracturing flows, the solution obtained in (Frigaard and Ryan, 2004) describes the flow pattern in the horizontal section of a vertical fracture. In (Fusi et al., 2012), the lubrication approximation was used to develop a generalized model for the 2D channel flow of a Bingham-like medium, which behaves as a linear elastic solid below the yield threshold.

A large number of papers deal with the Saffman–Taylor (S–T) instability accompanying displacement of particle-free fluids in a narrow plane channel or annulus, starting from the pioneering study by Muskat (1934) (in which the displacement of oil by water from oil-bearing sand was analyzed) and the classical study by Saffman and Taylor (1958), which provided a mathematical basis for the problem of instability at the interface between the fluids during displacement. It is found that the instability is triggered when a high-viscosity fluid is displaced by a low-viscosity one, and that fingers develop at the fluid–fluid interface. The growth rate of disturbances at the interface between immiscible fluids increases unboundedly with a decrease in the wavelength. In the presence of any "cut-off" mechanism, such as surface tension or miscibility, there is a certain maximum growing disturbance with a finite wave length. A comprehensive review of studies on the S–T instability in miscible and immiscible displacements of Newtonian fluids carried out in Hele-Shaw cells and porous media is presented in (Homsy, 1987).

The instability of the interface during the displacement of non-Newtonian fluids in a porous medium was analyzed for the first time by Pascal (1984a,b, 1986, 1988) in the framework of Muskat's model of piston-like displacement. The interface between power-law fluids with different power-law indices ($n_1 \neq n_2$) remains stable if the velocity of the displacement is below a certain threshold value, which is expressed via the consistency and power-law indices of the fluids. In other words, for specified parameters of power-law fluids with $n_1 \neq n_2$, there is a threshold displacing velocity, above which the instability occurs. If the power-law indices are identical, then the stability criterion for Newtonian fluids applies: the front remains stable if the consistency index of the displacing fluid is larger than that of the displaced one. In the case of viscoplastic fluids with both power-law and yield-stress behavior (Herschel–Bulkley rheology), the instability criterion involves the yield stresses of the fluids. Study (Coussot, 1999) deals with the instability of the front between Bingham fluids in a Hele-Shaw cell in the linear approximation. It is found that the dispersion relation is similar to that of Newtonian fluids, but involves the yield stress. Even at a very small injection velocity, a large amplification is gained by small-wavelength disturbances. Small fingers left behind in the beginning of the destabilization tend to stop, which is referred to as shielding effect.

As follows from the above review, none of the publications deals with the particular combination of physical effects, which is encountered in modeling the displacement of several yield-stress suspensions in a vertical plane channel, with application to hydraulic fracturing. To start filling the gap in the current body of knowledge, we present the development of a 2D width-averaged model for the flow of Bingham fluids in a narrow vertical channel (Hele-Shaw cell). The analysis covers various physical phenomena, such as the combined effect of gravitational slumping and viscous fingering, yielding and unyielding. The model proposed is suitable for the description of Bingham particle-free flows, or Bingham suspension flows, in which the particles are neutrally buoyant and their slip relative to the carrier fluid is negligible at the typical time scale (small inertia particles). It does not cover the effect of particle volume fraction on the yield stress of high-solid content fluid or on the particle settling velocity. These effects require further thorough theoretical and