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Two-dimensional computational fluid dynamical investigation of particle migration in rotating eccentric cylinders using suspension balance model

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ABSTRACT

Suspension balance and diffusive flux models have been developed to explain the particle migration phenomenon. Herein, we use the suspension balance model (SBM) to provide numerical validation of the particle migration in a concentrated suspension undergoing flow between rotating eccentric cylinders observed in the literatures. This study demonstrates that by implementing the mathematical model developed to explain the particle migration phenomenon, namely SBM into available commercial software such as COMSOL Multiphysics, one can readily explore the behavior of these systems. A two-dimensional finite element model of the SBM has been created in COMSOL Multiphysics. A set of transient conservation equations of bulk and particle-phase mass and momentum that included gravitational force effects were solved using the implicit time-stepping method of backward differentiation. The simulation method was validated using the available analytical solution for suspension in a circular Couette flow. The eccentricity ratio has been defined as $\varepsilon = e/(R_o - R_i)$, where R_o is the outer cylinder radius, R_i is the inner cylinder radius and e is the distance between the center of the inner and outer cylinders. The eccentricity ratio was analyzed, and it was shown that for the eccentricity ratio, $\varepsilon < 0.5$ and particle volume fraction, $\phi = 50\%$, the maximum concentration occurs along the circumference in the direction of the stationary cylinder. As ε increases and when $\phi = 50\%$, the maximum concentration appears along the horizontal mid-plane of the eccentric bearing in the wide-gap region. Further increases in the eccentricity ratio shift the maximum concentration region towards the rotating cylinder. The simulation results of the concentration distribution and the velocity profile compare well with the experimental data. This study provides a qualitative step forward in the application of computational fluid dynamics to suspension flows in various geometries and serves as a first step towards exploring the realistic three-dimensional modeling of dense suspensions in eccentric bearings as an example of general geometries.

Introduction

Shear-induced migration of particles in suspension flow is important for a wide variety of applications including food processing, ceramics and reinforced polymer composites, and the transport of slurries. Successful use of suspensions in engineering processes often requires that a flowing suspension be supplied at a specified location with a prescribed particle concentration. As flow specifications become more complex, the behavior of the flowing suspension needs to be better understood to effectively control particle concentrations throughout a process. The diffusive flux model by Phillips et al. (1992) and the suspension balance (SBM) model by Nott and Brady (1994) are the principal continuum models used to explain particle-migration phenomena. These models have mostly succeeded in predicting qualitative features of the migration process and quantitative steady-state velocity and concentration profiles in different flow situations. To evaluate the microstructure and rheology of concentrated suspensions, particle scale simulation methods such as Stoksian dynamics (Brady and Bossis, 1988) have also been utilized. However, due to large computational power and memory requirements, these simulations are limited to a few thousand particles.

Leighton and Acrivos (1987) first described shear-induced particle migration to explain anomalies observed in a Couette viscometer. However, their diffusive flux model was only applicable to simple planar and unidirectional flows. The extension of this model by Phillips et al. (1992) to more complex flows assumes that particle migration flux is driven by local particle-particle interaction

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frequency and viscosity variation and also assumes that the migration is isotropic.

Although the diffusive flux model successfully explains migration in channel and pipe flow, it fails in cone-and-plate and parallel plate torsional flows. This shortcoming is attributed to the assumption of isotropic behavior for interaction-induced flux in the original diffusive flux model. The diffusive flux model has been extended to more generalized applications and general conditions (Subia et al., 1998; Zhang and Acrivos, 1994; Rao et al., 2002; Fang and Phan-Thien, 1994; Fang et al., 2002). In addition, it has been extensively tested in Couette flow (Altobelli et al., 1991; Tetlow et al., 1998) and has been widely used in numerical simulations of suspension flow in various geometries (Subia et al., 1998; Zhang and Acrivos, 1994; Tetlow et al., 1998, Ahmed and Singh, 2011; Ingber et al., 2009).

The SBM was first proposed by Nott and Brady (1994) and is based on the conservation of mass and momentum for suspension and particle phases. In this model, averaging the mass and momentum conservation equations over the particle phase derives the particle phase transport equations. The suspension phase and particle phase transport equations incorporate a constitutive model for the suspension and particle stresses based on rheological theory and experimental data. This model provides a non-local description of suspension behavior in terms of the particle's velocity fluctuations. Nott and Brady (1994) showed that the diffusive flux model could be derived from the suspension balance model; therefore, the two models share the same physical origin.

In contrast to the model proposed by Nott and Brady (1994), when the particle phase stress was formulated with an isotropic particle phase pressure, Morris and Boulay (1999) showed the importance of anisotropy and normal stress difference effects for predicting migration in curvilinear flows. In Morris and Boulay's suspension balance approach, the particles migrate in response to spatial variation of the shear-induced normal stresses. The suspension normal stresses are directly modeled in terms of the rate of the strain field. Using this model and the finite volume method, Miller and Morris (2006) simulated the pressure-driven flow of a non-colloidal suspension in a two-dimensional channel and axisymmetric circular pipe geometries. Miller et al. (2009) have adapted the established shear-based rheological model of Morris and Boulay (1999) and have presented a frame-invariant formulation of the SBM for general geometries. The Miller et al. (2009) method does not involve solutions of the additional transport equation for suspension temperature, which accounts for the motions of the fluctuating particles, to describe non-local suspension stresses. They have also proposed that by considering the particulate phase as a continuum, some difficulties may arise at points where the shear rate approaches zero (e.g., at the centerline of the channel or pipe). Fluctuations in particle concentration alter the viscosity and shear rate at points on either side of the centerline, depending upon the parameter $\varepsilon = a/B$, where a is the particle radius and B is the half width of the channel. Therefore, a small but constant non-local contribution to the local shear rate was added to create a simplification of the spatial averaging approach of shear rate over a finite volume, as suggested by Morris and Boulay (1999). It should be noted that in the work of Miller et al. (2009), the parameter ε describes the size ratio; however, in this study this term is used for the eccentricity ratio.

Compared to the diffusive flux model, the SBM provides a more unified approach for particle migration. By including anisotropic particle stresses, the SBM successfully explains migration in curvilinear flows. Furthermore, the SBM in the form outlined by Morris and Boulay (1999), which includes the non-local shear rate concept (Miller and Morris, 2006; Miller et al., 2009), provides simple implementations to overcome the limitations of the diffusive flux model in certain situations. The diffusive flux model is also limited in its application to Brownian suspensions. Recent experimental studies have established particle migration in both concentrated and dilute regimes of Brownian suspensions (Frank et al., 2003; Semwogerere et al., 2007; Brown et al., 2009). To address the absence of experimental data on the measurement of normal stresses in Brownian suspensions, Frank et al. (2003) have extended the SBM for Brownian suspensions using the theoretical prediction of normal stresses (Brady, 1993; Brady and Vicic, 1995). Brown et al. (2009) have shown good agreement of the simulation results with their experimental measurements, using nuclear magnetic resonance imaging (NMRI), of shear-induced particle migration in Brownian suspensions. Kang et al. (2007) have attempted to include the particle flux resulting from the Brownian diffusion in the diffusive flux model. However, as this model requires several empirical parameters that are not available for Brownian suspensions, only a qualitative prediction of the migration behavior was presented. In this study, we consider non-Brownian suspension flow.

Both the suspension balance and diffusive flux models can be applied to the numerical simulation of general flows. Several studies have confirmed that the predictions of suspension balance and diffusive flux models closely match the experimental data for Couette flow.

To the author's knowledge, Phan-Thien et al. (1995) were the first to conduct experiments on eccentric bearings with an initial particle volume fraction of 0.50, in which NMR images of the particle distributions were obtained for an eccentricity ratio (ε) equal to 1/3, after 200, 800, and 14,000 revolutions of the inner cylinder. They also reported the (supposed) steady-state distribution of solid particles for $\varepsilon = 1/2$ together with a model for eccentric bearings using a finite volume approach and an unstructured grid. However, in their simulations they were unable to observe the recirculating flow in the wide-gap. Furthermore, the model predicted that the highest values of concentration occurred along the stationary cylinder in the wide-gap portion of the bearing. Later, Fang and Phan-Thien (1995) modeled the same problem using a finite volume method employing a structured grid. Their calculations revealed the recirculation region at a particle volume fraction of 0.20. Nonetheless, due to numerical instabilities, they were unable to obtain steady solutions for a particle volume fraction of 0.50. Thereafter, Fang et al. (2002) modeled the normal stress differences for the suspension balance formulation using a frame-invariant, flow-aligned tensor. The same flow-aligned tensor has also been used to reformulate the diffusive flux model. Using these flow-aligned tensor formulations, they were able to eliminate the main shortcomings of the original models in a unified manner. They also stated that the difference between the new normal stress differences model and the one proposed by Morris and Boulay (1999) (i.e., the original SBM) is that the diagonal elements of the directional tensor are dependent (i.e., $\lambda_1 = \lambda_2 = 2\lambda_3$). In addition, the measure of the particle fluctuating velocities (i.e., particle temperature) was preserved in their normal stresses model to introduce a measure of non-locality. Therefore, steady-state and transient simulations on various one-dimensional and two-dimensional flows were performed, for which experimental data existed, using finite-difference and finite-element schemes, respectively. In the present paper, the eccentric Couette flow of suspensions will be investigated based on the original SBM (presented by Morris and Boulay, 1999) in which $\lambda_1 = 1$, $\lambda_2 = 0.8$, and $\lambda_3 = 0.5$ and also using computational fluid dynamics (CFD) methods. Furthermore, we explore the effect of one of the critical parameters in this problem, i.e., the eccentricity ratio, ε, on the flow.

Subia et al. (1998) experimentally and numerically investigated the behavior of suspensions of large-particle, non-Brownian and