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Dispersion and deposition of ellipsoidal particles in a fully developed laminar pipe flow using non-creeping formulations for hydrodynamic forces and torques

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ABSTRACT

In this study, dispersion and deposition of ellipsoidal particles in fully developed laminar pipe flows were analyzed using new correlations for hydrodynamic forces and torques for non-creeping flow conditions. In these simulations, flow Reynolds numbers up to 400 were considered and the fiber Reynolds numbers in the range of 0.01–6 were studied. The deposition efficiency of ellipsoidal fibers of different sizes and aspect ratios in laminar pipe flows were simulated for various flow and particle Reynolds numbers. A new empirical model for estimating fiber deposition efficiency for finite particle Reynolds numbers was developed. The simulation results were compared with earlier theoretical and numerical studies, as well as the new empirical equation, and good agreement was found. Particular attention was given to the effect of fiber size and aspect ratio on dispersion and deposition of particles under various flow conditions for fully developed laminar pipe flow.

Introduction

Transport, dispersion and deposition of particles suspended in a fluid are encountered in many areas of fluid engineering such as liquid fuel spray injection into the combustion chamber, gas–solid flows in pneumatic conveyers, sediment transport in rivers, and transport of particles in the human respiratory system.

Most of the earlier studies of particle-laden flows treated the particles as spheres although the vast majority of industrial and natural particles are non-spherical and have irregular shapes. For spherical particles, analysis of their translational motions provides a complete description of their motions. Prediction of the transport and deposition of non-spherical particles, however, is more complicated due to the coupling of particle translational and rotational motions. In order to analyze the trajectory of non-spherical particles, the coupled translational and rotational equations of motion should be considered simultaneously. In many cases non-spherical particles are modeled using equivalent diameter spheres, and/or with the use of a shape factor. A commonly used shape factor for non-spherical particles is sphericity, SP, Wadell (1934) which is defined as the ratio of surface area of the equivalent volume sphere to the surface area of the non-spherical particle. For spherical particles, sphericity is one (SP = 1). The sphericity, however, does not capture information about the orientation of highly elongated non-spherical particles. Rosendahl (2000) and Holzer and Sommerfeld (2008) suggested using sphericity for different directions.

Detailed reviews of the existing methods for simulation of non-spherical and elongated particles may be found in the studies of Fan and Ahmadi (1995), Soltani and Ahmadi (2000), Zhang et al. (2001), Mando and Rosendahl (2010) and Dastan et al. (2014). Transport and deposition of elongated fibers in pipes and channels were studied by Kvasnak and Ahmadi (1996), Fan and Ahmadi (1995) and Zhang et al. (2007). Recently, Mortensen et al. (2008a, 2008b), Marchioli et al. (2010), Shanley and Ahmadi (2011), Tian et al. (2012), Tian and Ahmadi (2013) and Feng and Kleinstreuer (2013) studied the motion of fibrous particles in developing and fully developed tubular and duct flows. Results of these studies revealed that in the near wall region of pipes and ducts, ellipsoidal particles tend to align with the mean flow direction, and alignment becomes more pronounced as the fiber aspect ratio increases. However, far from the wall and in the core region of the pipe or duct the isotropic fiber orientation is observed. More recently, Tavakol et al. (2015) analyzed dispersion and deposition of fibrous particles in various turbulent fields. They incorporated the role of

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turbulence fluctuations using proper stochastic models and obtained satisfactory results for dispersion and deposition of fibers in fully developed pipe and duct flows.

In the earlier works, Ahmadi and co-workers used the Euler–Lagrange method and solved the coupled translational and rotational motions of ellipsoidal in laminar and turbulent flows. The use of Euler's quaternions to account for the fiber orientation was also suggested in these earlier efforts.

In the past studies, creeping flow formulations for hydrodynamic forces and torques acting on the fibers were used in the analysis. Recently, new correlations for the hydrodynamic drag and torque coefficients for non-creeping flow conditions have been reported in the literature. The Lattice Boltzmann (LBM) study of Holzer and Sommerfeld (2009) and the direct numerical simulation (DNS) of Zastawny et al. (2012) provided useful data and correlations for the drag, lift and torque coefficients for some non-spherical particles. van Wachem et al. (2015) used their proposed correlations in the study of Zastawny et al. (2012) to predict the behavior of the interacting non-spherical particles with large Stokes numbers in a turbulent flow. They showed that the non-spherical particles tend to align their longest axis perpendicular to the local flow.

The present study is concerned with evaluating the importance of non-creeping flow drag and torque acting on ellipsoidal particles (ellipsoids of revolution). The hydrodynamic forces and torques obtained from the study of Holzer and Sommerfeld (2009) and the study of Zastawny et al. (2012), which are for finite particle Reynolds numbers, are used for simulating the motion of fibrous particles in a fully developed laminar pipe flow. For analysis first the creeping flow formulation and solution methodology are reviewed. This is followed with the description of new correlations for the hydrodynamic forces and torques at finite particle Reynolds numbers in a non-creeping flow regime. The simulation results for dispersion and deposition of fibrous particles of various sizes and aspect ratios in the laminar flow are presented and discussed. A new modified empirical model for fiber deposition in laminar pipe flows was developed. The simulation results for fiber deposition efficiencies are compared with the empirical equations and good agreement are found.

Formulations

Creeping flow formulation

The motion of a non-spherical particle is described by the coupled translational and rotational equations of motion. Translational motion is governed by the balance of linear momentum for the particles. Rotational motion is governed by the equations representing the balance of angular momentum. To analyze motion of an ellipsoidal fiber, it is found convenient to introduce three Cartesian coordinate systems as shown in Fig. 1a (Goldstein, 1980; Fan and Ahmadi, 1995). The (x, y, z) denotes the inertial coordinate system, while $(\hat{x}, \hat{y}, \hat{z})$ coordinate system is the particle frame with its origin located at center of mass of the fiber and with the $\hat{z}$ axis along the major axis of the fiber. The third coordinate system $(\hat{\hat{x}}, \hat{\hat{y}}, \hat{\hat{z}})$ is called the fiber co-moving frame, and it is attached to the fiber's center of mass with its axes parallel to the inertial coordinate system. These coordinate systems and schematics of an ellipsoidal particle are shown in Fig. 1.

The inertial coordinate system is used to describe the fiber translational motion, while the rotational equations of motion are stated in the particle coordinate system $(\hat{x}, \hat{y}, \hat{z})$. The transformation from one coordinate system to the other is described by means of the Euler angles Φ, θ, and ψ. In Fig. 1b, Φ denotes the incidence angle, the angle between fiber's major axis and relative velocity vector.

Rotations by means of Euler's angles transform the coordinate systems from co-moving frame to particle frame. This transformation describes the Goldstein (1980) x-convention rotation. That is, for any vector the coordinate transformation is given as:

$$\vec{\hat{x}} = A\vec{\hat{\hat{x}}} \tag{1}$$

Here A denotes the transformation matrix defined in terms of Euler angles as

$$A = \begin{bmatrix} \cos\psi\cos\varphi - \cos\theta\sin\varphi\sin\psi & \cos\psi\sin\varphi + \cos\theta\cos\varphi\sin\psi & \sin\psi\sin\theta \\ -\sin\psi\cos\varphi - \cos\theta\sin\varphi\cos\psi & -\sin\psi\sin\varphi + \cos\theta\cos\varphi\cos\psi & \cos\psi\sin\theta \\ \sin\theta\sin\varphi & -\sin\theta\cos\varphi & \cos\theta \end{bmatrix} \tag{2}$$

As reported by Fan and Ahmadi (1995) and Dastan and Abouali (2013), due to the potential singularity of the transformation matrix, it is advantageous to use the quaternions $(\varepsilon_1, \varepsilon_2, \varepsilon_3, \eta)$ instead of Euler's angle in the transformation matrix. That is,

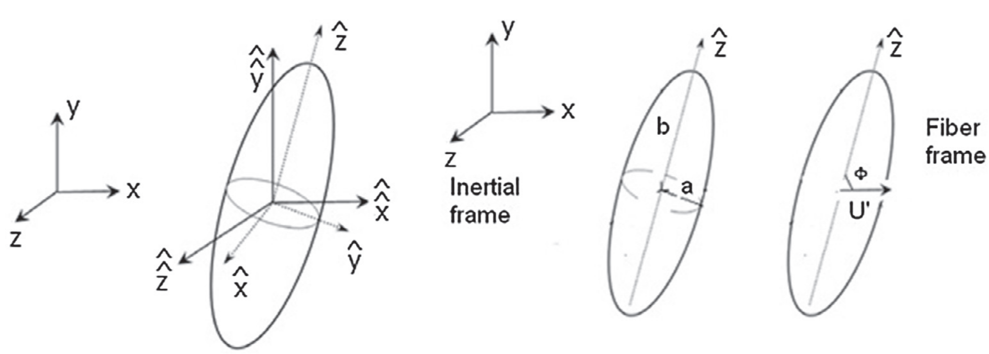
$$A = \begin{bmatrix} 1-2(\varepsilon_2^2+\varepsilon_3^2) & 2(\varepsilon_1\varepsilon_2+\varepsilon_3\eta) & 2(\varepsilon_1\varepsilon_3-\varepsilon_2\eta) \\ 2(\varepsilon_1\varepsilon_2-\varepsilon_3\eta) & 1-2(\varepsilon_1^2+\varepsilon_3^2) & 2(\varepsilon_3\varepsilon_2+\varepsilon_1\eta) \\ 2(\varepsilon_1\varepsilon_3+\varepsilon_2\eta) & 2(\varepsilon_3\varepsilon_2-\varepsilon_1\eta) & 1-2(\varepsilon_2^2+\varepsilon_1^2) \end{bmatrix} \tag{3}$$

The Euler quaternions are related to Euler's angles. That is,

$$\varepsilon_1 = \cos\frac{\varphi-\psi}{2}\sin\frac{\theta}{2} \tag{4}$$

$$\varepsilon_2 = \sin\frac{\varphi-\psi}{2}\sin\frac{\theta}{2}$$

$$\varepsilon_3 = \sin\frac{\varphi+\psi}{2}\cos\frac{\theta}{2}$$



(a) Three coordinate systems for describing fiber motion.

(b) Geometry of fiber and angle of incidence Φ in the fiber frame.

Fig. 1. Fiber geometry and different coordinate systems.