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Characteristics of stratified laminar flows in inclined pipes

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ABSTRACT

Exact solutions for laminar stratified flows in inclined pipes are presented. These include all possible configurations of stratified flows with concave and convex interfaces. The exact solution is used to study the effect of the flow geometry and pipe inclination on the holdup and pressure gradient. In particular, the possibility of reducing the pressure gradient of a viscous fluid in upward inclined flows by adding a lubricating phase is investigated. It is shown that independently of the density of the lubricant, namely, whether it is lighter or heavier than the viscous fluid, the effect of hydrostatic pressure gradient always adversely affects the possibility to reduce the pumping requirement for the flow of the viscous phase. In addition, the countercurrent flow limits and the boundaries of the multiple solution regions in concurrent inclined flow are identified and discussed. The susceptibility of the system to the Ledinegg instability when using separate pumps for each of the fluids is also examined.

Introduction

Stratified flow (STF) is considered a basic flow pattern in horizontal or slightly inclined gas–liquid and liquid–liquid systems of a finite density difference, since for some range of sufficiently low flow rates, the two phases tend to segregate. The stratified flow may occupy the entire pipe, or parts of its length (e.g., the tail of long slugs). Counter-current stratified flow is encountered in the process industry, in various mass transfer and direct contact heat transfer systems, and is feasible only in inclined systems (Ullmann et al., 2003a).

In principle, exact analytical solutions for the two-dimensional velocity profiles and shear stress profiles in pipe flow can be obtained only for fully developed laminar flows. Such solutions are of practical significance mainly for studying two-phase flow in small diameter pipes or liquid–liquid flows. The range of operational conditions where stratified flow can be established in min/micro channels was found to be affected by various factors, such as the inlet device (and premixing), tube inclination, surface wetting conditions due to tube surface material and start-up procedure (e.g., Dreyfus et al., 2003; Matsumoto et al., 2007; Salim et al., 2008; Wang et al., 2012; Ami et al., 2012; Mehta and Banerjee, 2014). Experiments in gas–liquid flows indicated that the stratified flow region shrinks in mini channels (e.g., Shao et al., 2009; Chinnov and Kabov, 2006; Zhao et al., 2013). It was observed to be limited to very low liquid flow rates and low liquid cut, which is also supported by modeling of flow pattern maps in minichannels (Ullmann and Brauner, 2007). It is worth noting that it is difficult to detect the flow pattern and the shape of the interface between the phases by side view observations commonly used in flow pattern studies. In particular, it may be difficult to distinguish between stratified and eccentric annular flows. The interface configuration can be determined by applying sophisticated insitu measurement techniques and methods for reconstruction of the interface (e.g., Angeli and Hewitt, 2000; Patel and Garimella, 2014; Edomwonyi-Otu and Angeli, 2015) and by CFD simulations (e.g., Ong et al., 1994), where stratified flows with curved interface have been obtained. The development of accurate models for predicting the stratified flow characteristics may advance also the models for predicting the effects of the above parameters on the operational region where stratified flow may be established.

The exact solutions for laminar stratified flows are also needed as benchmark problems for testing the validity of numerical methods for solving general two-phase separated flows (e.g., Ng et al., 2002; Berthelsen and Ytrehus, 2004) and for testing closure relations for simplified one-dimensional two-fluid models (e.g., Ullmann et al., 2004; Ullmann and Brauner, 2006). The latter are widely used for engineering design of two-phase pipelines. However, two-fluid models may yield poor predictions in inclined co-current and counter-current flows, as the commonly used closure relations for the wall and interfacial shear stresses, which

Abbreviations: STF, Stratified flow; CAF, Core-annular flow; FE, Fully eccentric.

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are based on single-phase theory/correlations, are not representing correctly the fine balance between the gravity body forces and the viscous shear in inclined flows.

For inclined pipes analytical solutions are available in the literature only for laminar fully-developed STF with a plane interface (Biberg and Halvorsen, 2000; Goldstein, 2002; Ullmann et al., 2004). However, stratified flow with a plane interface is typical to gravity-dominated systems. In surface tension (σ) dominated systems (e.g., microgravity, capillary tubes, and in liquid–liquid systems, namely: low g, small D, or small density difference $\Delta\rho$, respectively), the *Eotvös* number, $Eo_D = \Delta\rho g D^2/\sigma$ and the fluids/wall contact angle, α are important parameters for determining the interface shape in smooth-stratified flows (Brauner et al., 1996b; Gorelik and Brauner, 1999; Liu et al., 2008). The wetting liquid tends to spread over the tube wall resulting in a curved (convex or concave) interface (see Fig. 1). Indeed, various stratified flow configurations can be stabilized in micro-channels by using specially designed tubes with their inner surface coated partially by hydrophilic and partially by hydrophobic materials (e.g., Yamasaki et al., 2010; Salim et al., 2008). In fact, the possible stratified flow configurations may extend from fully eccentric core of the upper phase to fully eccentric core of the lower phase. Obviously, the associated variations in the fluid–wall and fluid–fluid contact areas may have prominent effects on the pressure drop and transport phenomena. However, the solutions available in the literature for stratified flow with curved interfaces are restricted to horizontal flows (Bentwich, 1964; Brauner et al., 1996a; Rovinsky et al., 1997), and cannot be applied to inclined systems in case of different densities of the two phases. Multiple holdups and pressure drops can be obtained for specified operation conditions in co-current and counter-current inclined flows, which are relevant in practical applications (Ullmann et al., 2003a,b). Identifying the multiple holdup regions and their variation with the stratified flow configuration is of importance for predicting and controlling the system operation. In this paper a complete set of analytical solutions is presented for horizontal and inclined stratified flows that covers the entire range of stratified flow configurations with constant interfacial curvature, which extends from fully eccentric core of the lower phase, to fully eccentric core of the upper phase. The solutions obtained can be used to study the effects of the pipe inclination, the interface curvature on the characteristics of counter current and co-current flows. The pronounced effects of the interfacial curvature are demonstrated and discussed. Practical aspects are addressed, such as the possibility of reducing the pressure gradient of a viscous fluid by adding a lubricating phase, and the susceptibility of the system to the Ledinegg instability. The countercurrent flow limits and the boundaries of the multiple solution regions in concurrent inclined flow are identified and discussed.

Exact solutions for laminar stratified flows

Velocity profiles

Given the location of the fluids interface, the 2-D velocity profiles, $u(x,y)$ in a steady and fully developed axial laminar pipe flow of two separated phases are derived from the solution of the Navier–Stokes equations (in the flow direction, z):

$$\mu_j \nabla^2 u_j = \frac{\partial p}{\partial z} - \rho_j g \sin\beta; \quad j = H, L \tag{1}$$

where (H,L) denote the heavy (lower) and the light (upper) layers respectively, p is the pressure, β is the pipe inclination to the horizontal, and μ,ρ are the viscosity and density of the phases, respectively. The required boundary conditions follow from the no-slip condition at the pipe wall and continuity of the velocities and tangential shear stresses across the fluids' interface.

Exact analytical solutions for Eq. (1) can be obtained in the bipolar coordinate system for stratified flows in the case of constant interface curvature (represented by ϕ^* in Fig. 1). The assumption of constant curvature of the interface is consistent with the exact solution for the interface shape in the two extreme cases of gravity dominated systems, $Eo_D \gg 1$, or surface tension dominated systems, $Eo_D \ll 1$ (Gorelik and Brauner, 1999). Obviously, any deviation from a constant curvature shape of the interface in $Eo_D \ll 1$ systems would result in transverse flows due to non-uniform pressure jump across the curved interface, which is inconsistent with the condition of fully developed (axial) flow. In-between the two extremes of the Eo_D, it was shown in Gorelik and Brauner (1999) that using the model of Brauner et al. (1996b) for the characteristic (constant) interface curvature yields a fair approximation to the exact interface shape for the entire range of Eo_D.

In the bipolar coordinate system (ϕ, ξ), (e.g., Moon and Spencer, 1971) the pipe perimeter and the interface between the fluids are iso-lines of coordinate ϕ, so that the upper section of the tube wall bounding the lighter phase is represented by ϕ_0, while the bottom of the tube, bounding the denser phase, is represented by $\phi_0 + \pi$. The interface coincides with the curve of $\phi = \phi^*$. Thus, the two-phase domains map into two infinite strips in the domain defined by: $-\infty < \xi < \infty, \phi^* > \phi > \phi_0$ for the upper phase, and $-\infty < \xi < \infty, \phi_0 + \pi > \phi > \phi^*$ for the lower phase. The relations between ϕ_0, ϕ^* and the geometrical variables (e.g., flow areas, wetted perimeters) are given elsewhere (Brauner et al., 1998). A plane interface corresponds to a constant curvature arc, $\phi^* = \pi$. In this

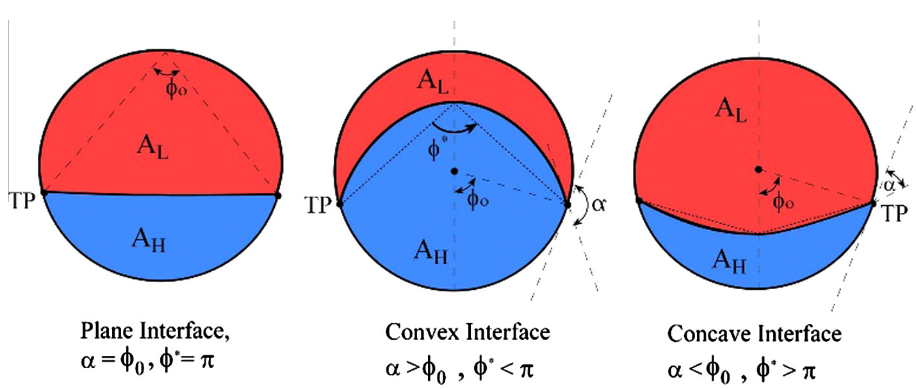

Fig. 1. Schematic description of the stratified flow configurations and parameters.