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Stability of a rivulet flowing in a microchannel

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A B S T R A C T

A novel microfluidic technique has been recently proposed to produce quasi-monodisperse collections of microbubbles with a controlled size. In this technique, a gaseous stream is injected through a T-junction into a microchannel transporting a liquid current. The gas adheres to a hydrophobic strip printed on the channel surface. When the gas and liquid flow rates are set appropriately, a gaseous rivulet flows over that strip. The rivulet breaks up downstream due to a capillary pearling instability, which leads to a quasi-monodisperse collection of microbubbles. Motivated by this application, we here analyze the stability of both gas and liquid rivulets coflowing with a current in a quadrangular microfluidic channel. The results essentially differ from those of cylindrical jets because the contact-line-anchorage condition affects fundamentally the rivulet's instability nature. The temporal stability analysis shows that the rivulet becomes unstable not only for (unperturbed) contact angles larger than 90° (as can be expected) but also for values smaller than that angle. Interestingly enough, the maximum growth factor exhibits a non-monotonic dependence with respect to the Reynolds number (i.e., the viscosities). In fact, there are intervals of that parameter where the fluid system becomes unstable, while all the perturbations are damped outside that interval. The gaseous rivulet does not stabilize as the Reynolds number decreases, which means that it can be unstable even in the Stokes limit and for contact angles less than 90°. In addition, the stability of a flowing liquid rivulet is not determined by its contact angle exclusively (as occurs in the static case), but by the Reynolds number as well. Liquid rivulets with contact angles less than 90° can be unstable for sufficiently high Reynolds numbers.

Introduction

When a jet touches and sticks to a solid wall, it forms a rivulet. This fluid configuration plays an important role in a number of industrial applications. Here, we just mention some examples. The interfacial shear caused by the overlying gas in heat exchangers significantly affects the performance of these devices. Rivulets driven by the shear force exerted by the surrounding air are frequently considered when studying the icing of aircraft components. When gravity is the driving force, the rivulet flow is exploited in trickle bed reactors and structured packings. Rivulets are also formed to producing coated surfaces for varied applications. In this case, one may be interested either in the formation of very uniform coatings or in the generation of certain fluid patterns. While in the former case the aim is to quench the instability mechanisms, unstable shapes are exploited in the latter one to producing the desired pattern.

In principle, fluid rivulets can be produced in microfluidic devices by printing micrometer lyophilic/lyophobic stripes on the channel surface. A number of methods can be used for this purpose, including vapor deposition through grids, elastomer stamps, domain formation in Langmuir–Blodgett monolayers, and photolithography of amphiphilic monolayers. These chemical ducts can only be created if the contact angles characterizing the lyophilic and lyophobic surfaces verify certain conditions (Brinkmann and Lipowsky, 2002). The use of chemical ducts in microfluidics prevents from clogging by solute particles, such as colloids or large bio-polymers, which constitutes an important advantage. Based on this idea, Herrada et al. (2013) have recently proposed a microfluidic technique to produce quasi-monodisperse collections of microbubbles in a controlled manner. In this technique, a gaseous stream is injected through a T-junction into a channel transporting a liquid current. A hydrophobic strip is printed on one of the channel surfaces, and thus the gas stream forms a rivulet over that strip. If the rivulet is convectively unstable (Huerre and Monkewitz, 1990), it breaks up downstream due to a capillary pearling instability, which leads to a quasi-monodisperse collection of microbubbles that can be much smaller than the channel size. For the sake

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of illustration, Fig. 1 shows numerical simulations of the fluid configuration analyzed by Herrada et al. (2013). Image (a) corresponds to the case in which the gaseous rivulet does not form, while images (b) and (c) show a convectively unstable and stable rivulet, respectively.

The linear stability of liquid rivulets has been frequently studied over the last two decades. It crucially depends on the behavior of the triple contact lines (Young and Davis, 1987). If they are allowed to move, the rivulet suffers from pearling instability in any case (analogously to what happens in jets). If they are perfectly pinned, then a static rivulet is unconditionally stable for an unperturbed contact angle lower than 90°, while there is a range of unstable wavenumbers if the contact angle exceeds that threshold. These results were originally obtained for an infinite rivulet resting on a flat surface (Davis, 1980; Brown and Scriven, 1980), and subsequently extended to more complex equilibrium configurations, such as rivulets of finite length (Gau et al., 1999; Brinkmann and Lipowsky, 2002) or rivulets lying on substrates of varied shapes (Roy and Schwartz, 1999).

The stability of flowing rivulets with anchored contact lines has also been considered by several authors. The lubrication and thin-film approximations have been used to calculate the basic flows driven not only by the gravitational force (Perazzo and Gratton, 2004; Paterson et al., 2013) but also by other factors, such as a prescribed uniform transverse shear stress at its free surface (Sullivan et al., 2012). Under those approximations, rivulets flowing over a vertical plane (Mechkov et al., 2008) and under a sloping plate (Benilov, 2009) have proved to be stable as long as their triple contact lines are fixed. Weiland and Davis (1981) have shown that shallow rivulets with pinned contact lines flowing down over a vertical surface become unstable if the driving force exceeds a critical value which increases as the Reynolds number decreases. Koplik et al. (2006) studied the stability of nano-rivulets driven by gravity from both the linear stability analysis of the Navier–Stokes equations and molecular dynamics simulations. The configurations analyzed were stable if and only if the contact angle was lower than 90°.

Linear stability analysis provides quantitative predictions for the size of the droplets resulting from the rivulet breakup. Diez et al. (2009) have found that the distance between the drops that form during the nonlinear evolution is essentially determined by the wavelengths predicted by the linear approximation. Herrada et al. (2013) have also shown a good agreement between the droplet size obtained from the linear stability analysis and the simulations of the full Navier–Stokes equations.

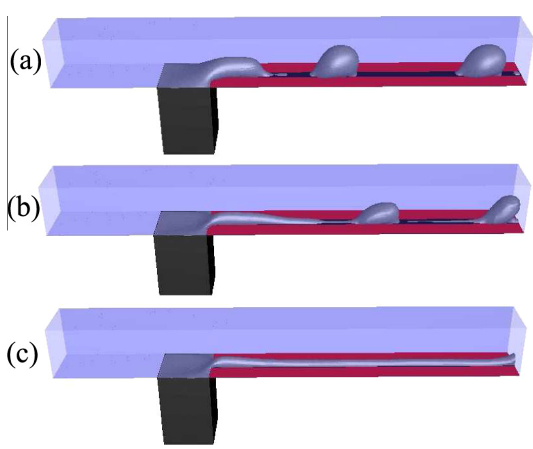

Fig. 1. Flow snapshots for an air-ethanol rivulet (Herrada et al., 2013). The images correspond to three different regimes depending on the gas Q_g and liquid Q_l flow rates: (a) bubbling ($Q_g = 3.6$ ml/h and $Q_l = 36$ ml/h), (b) convectively unstable rivulet ($Q_g = 1.8$ ml/h and $Q_l = 72$ ml/h), and (c) stable rivulet ($Q_g = 0.09$ ml/h and $Q_l = 72$ ml/h).

Several authors have studied the rivulet's stability by determining when it is energetically favorable for the rivulet to break up into sub-rivulets. Schmuki and Laso (1990) found that this is the case for thin rivulets on a sloping plate under certain conditions. The combined action of a body force and a uniform longitudinal shear stress have been examined from the energy approach too (Myers et al., 2004; Saber and El-Genk, 2004; Wilson and Duffy, 2005; Wilson et al., 2011). For instance, Wilson and Duffy (2005) determined the conditions for which a thin rivulet on a inclined substrate and in the presence of a prescribed uniform longitudinal shear stress splits into sub-rivulets. It must be noted that it is not clear how linear stability analysis results compare with those derived from the energy method, because small disturbances are not necessarily capable of destabilizing equilibrium states corresponding to local energy minima.

To the best of our knowledge, the stability of a gaseous rivulet has been considered only by Herrada et al. (2013). While that work was focused on the proposal of a novel microfluidic technique to producing microbubbles, a preliminary temporal stability analysis of the Navier–Stokes equations was conducted to prove the suitability of the proposed method. This analysis led to the same conclusion as that for both static (Davis, 1980; Brown and Scriven, 1980) and flowing (Koplik et al., 2006) liquid rivulets: configurations with perfectly anchored triple contact lines and contact angles less than 90° are stable. As mentioned above, the linear analysis provided good predictions for the microbubble size, particularly accurate close to the rivulet stability limit.

In this paper, we will extend the analysis mentioned above by considering both gas and liquid rivulets, and by extending the explored parameter region. The new results show that fluid rivulets can be unstable for contact angles smaller than 90° too. Contrarily to what one might expect, the maximum growth factor exhibits a non-monotonic dependence with respect to the Reynolds number, so that there are intervals of that parameter where the rivulet becomes unstable. As will be explained, this implies that a certain basic flow can become unstable when viscosity increases while the rest of governing parameters remain constant.

Linear stability analysis

The governing equations

Consider an infinite rivulet of density ρ_1 and viscosity μ_1 moving over a strip of width w (Fig. 2). The rivulet coflows with an outer stream of density ρ_2 and viscosity μ_2 within a quadrangular

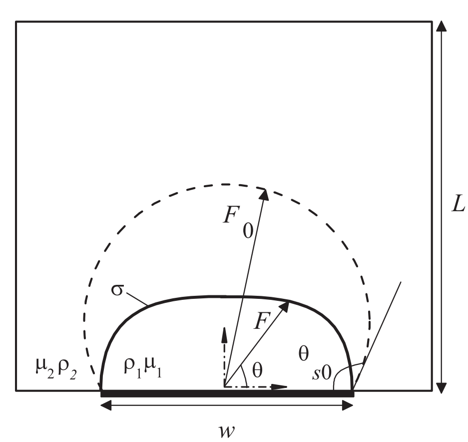

Fig. 2. Sketch of a cross section of the fluid configuration considered. The dashed (solid) line represents the unperturbed (perturbed) rivulet shape.