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Rising motion of a swarm of drops in a linearly stratified fluid

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ABSTRACT

Direct numerical simulations of a swarm of deformable drops rising in density stratified fluids are presented at intermediate Reynolds numbers. All flow scales are fully resolved using front-tracking/finite-volume method. The average rise velocity and velocity fluctuations of the swarm are reduced in the presence of density stratification. The isotropy in velocity fluctuations is enhanced as the volume fraction increases. The higher likelihood of the cluster formation is illustrated in the presence of density stratification and is explained by quantitative assessment of the microstructure using radial and angular pair probability distribution functions. The combined effect of the drop deformability and density stratification on the average deformation of the drops is investigated.

Introduction

In oceans and lakes, vertical variation of water temperature or salinity results in the generation of vertical density layering in the water column. The rising motion of bubbles in oceans and lakes (Bayareh et al., 2013), bubble mixers used for aeration of lakes and reservoirs (Hill et al., 2008) and motion of drops during oil spills (Blumer et al., 1971) are few examples of important processes that are being affected by density stratification. Oil spills can cause extensive hazards to marine and wildlife habitats as well as fishing and tourism industry (Juhasz, 2012). The ocean density stratification is known as one of the main factors in trapping of the oil plume and dispersed drops (Socolofsky and Adams, 2003; Camilli et al., 2010). Understanding the effect of stratification on the rising motion of the swarm of drops is necessary for accurate estimation of the rising time, dispersion of the oil, and consequently biodegradation. Recent studies have shown that the motion of a single drop through either a sharp or continuous density stratification is substantially affected by stratification. For a drop settling through an interface between two fluids of different densities, Blanchette and Shapiro (2012) reported a significant reduction of the settling velocity at the interface in the absence of the Marangoni effects and even the reversal of the motion of the drop in the presence of the Marangoni effects. The study of the settling dynamics through a sharp density interface provides important insights about the physics of the motion of a bubble/drop in stratified fluids. However, the size of bubbles/drops in aquatic environments are generally much smaller than the length scales of density gradients in the water column and thus a more realistic physical model in the natural environment is represented by a linear stratification. It has been recently found by Bayareh et al. (2013) that the presence of a linear density gradient results in a notable drag enhancement of a rising drop and subsequently extends the travel time of the drop in the water column by up to 30%.

The dynamics of the settling and drag enhancement of rigid particles in stratified fluids has been reported both experimentally and numerically (Srdić-Mitrović et al., 1999; Torres et al., 2000; Yick et al., 2009; Doostmohammadi et al., 2014; Doostmohammadi and Ardekani, 2014). When the viscous forces dominate the inertial effects, the drag enhancement is due to the entrainment of a light fluid behind the rigid particle (Yick et al., 2009; Doostmohammadi et al., 2012), while in a strong inertial regime the collapse of rear vortices behind the particle results in the higher resistance to the vertical motion of rigid particles (Torres et al., 2000). For a pair of rigid particles settling in a linearly stratified fluid, Doostmohammadi and Ardekani (2013) quantified the role of stratification on the interaction between the two particles. Authors showed that for a pair of particles settling side-by-side, unlike a homogeneous fluid, stratification results in the attraction between the particles. In addition, prolonged collision time was reported for in-tandem settling of a pair of particles in stratified fluids compared to the homogeneous counterpart. For a cloud of particles in a stratified fluid, Luketina and Wilkinson (1994) showed the entrainment of the ambient fluid by the particle cloud up to a maximum depth where the particle fall out. Experiments of Hussain and Narang (1984) demonstrated the formation of a double plume

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structure of bubbly flows when they interact with a stratified fluid. Socolofsky and Adams (2003) then showed that the buoyancy effects slow down the vertical intrusion of the plume of bubbles and result in a horizontal intrusion of a plume that constantly entrains the surrounding fluid. Despite the recent studies of the motion of a single drop in stratified fluids and studies of the interaction of plumes of bubbles with stratified fluids, the research on the interactive motion of swarms of deformable particles/drops in the presence of the density stratification is virtually non-existent.

In this study, we numerically investigate the effects of density stratification on the ascending motion of a swarm of drops. We particularly focus on the spatial distribution of the drops and cluster formation in order to characterize the microstructure of the swarm. In addition, the effect of stratification on pseudo-turbulence properties of the flow is presented.

Governing equations

The migration of a swarm of drops in an incompressible, linearly stratified fluid is governed by the following equations (Bayareh et al., 2013):

$$\nabla \cdot \mathbf{u} = 0, \tag{1}$$

$$\rho \frac{D\mathbf{u}}{Dt} = -\nabla p + (\rho - \bar{\rho})\mathbf{g} + \nabla \cdot \mu(\nabla \mathbf{u} + (\nabla \mathbf{u})^T) + \int \sigma \kappa' \hat{\mathbf{n}} \delta^\beta (\mathbf{x} - \mathbf{x}')\, dA', \tag{2}$$

$$\frac{DT}{Dt} = \kappa \nabla^2 T, \tag{3}$$

where $\mathbf{u}$ is the velocity vector, t the time, p the pressure, and $\mathbf{g}$ is the acceleration of gravity. The local density and viscosity of the fluid are ρ and μ, respectively. The last term in Eq. (2) represents the interfacial tension between the continuous and dispersed phases and it is evaluated at point $\mathbf{x}$. κ' is twice the mean curvature of the interface of the drop, $\hat{\mathbf{n}}$ is the unit vector normal to the interface, dA' is the surface element at the interface of the drop, δ^β is the three-dimensional delta function which is discontinuous at $\mathbf{x}'$, located on the interface, $\bar{\rho} = \frac{1}{L^3}\int_V \rho\, dV$ is the mean density over the entire computational domain, κ is the thermal diffusivity coefficient, and T is the temperature. In driving Eq. (3), we have assumed that the thermal diffusivity and conductivity coefficients in the dispersed and continuous phases are uniform and equal (Bayareh et al., 2013). Eqs. (2) and (3) are coupled by assuming a linear relation between the density and the temperature, i. e., $\rho = \rho_0(1 - \beta(T - T_0))$, where β is the coefficient of thermal expansion and the reference density and reference temperature are shown by ρ_0 and T_0, respectively. Fluid properties in and out of the drops are distinguished by defining a color function α which is zero inside and unity outside the drops. Thus $\rho_0 = \alpha\rho_{f_0} + (1-\alpha)\rho_{d_0}$, $\mu = \alpha\mu_f + (1-\alpha)\mu_d$ and $\beta = \alpha\beta_f + (1-\alpha)\beta_d$ represent fluid properties in the entire domain, where subscript f refers to the continuous phase and d to the dispersed phase, respectively.

The physics of the motion of the swarm of drops in a linearly stratified fluid can be characterized by a number of dimensionless parameters. The Archimedes number $Ar = gd^3\rho_{f_0}(\rho_{f_0} - \rho_{d_0})/\mu_f^2$ represents the ratio of gravitational force to the viscous force acting on a drop, where d denotes the diameter of a spherical drop. We use the Eötvös number $Eo = (\rho_{f_0} - \rho_{d_0})gd^2/\sigma$ to characterize the deformability of drops. The stratification of the fluid is characterized by the Froude number $Fr = W/(Nd)$, where $N = (\gamma g/\rho_{f_0})^{1/2}$ is the buoyancy frequency and γ is the background density gradient in the water column. We define the reference velocity W based on Hadamard–Rybczynski velocity (Hadamard, 1911; Rybczynski, 1911):

$$W = \frac{1}{6}\frac{g(\rho_{f_0} - \rho_{d_0})d^2}{\mu_f}\frac{\mu_f + \mu_d}{2\mu_f + 3\mu_d} \tag{4}$$

which corresponds to the settling velocity of an isolated drop in an unbounded homogeneous fluid in the Stokes regime. Unless otherwise stated, the velocity is scaled with W and time is scaled with $\tau = d/W$. The ratio of the diffusivity of the momentum ν_f to the diffusivity of the stratifying agent is represented by the Prandtl number $Pr = \nu_f/\kappa$. The volume fraction of N_d number of drops in a periodic box of length L is defined as $\phi = N_d\pi d^3/6L^3$. The ratios of material properties $\eta = \rho_{d_0}/\rho_{f_0}$, $\lambda = \mu_d/\mu_f$ and $B = \beta_d/\beta_f$ are other dimensionless parameters of the problem. In order for the swarm to reach a steady state condition, we set $\rho_{f_0}\beta_f = \rho_{d_0}\beta_d$, so that the drop density reduces as it enters warmer fluid layers. As a result, the spatial variation of the temperature inside and outside the drops become identical and drops eventually reach a statistically steady rise velocity (Bayareh et al., 2013).

The rise Reynolds number $Re_W = W_s d/\nu_f$ is calculated *a posteriori* based on the statistically steady-state average slip velocity of the swarm of drops W_s. The slip velocity is defined as the relative velocity between the dispersed and continuous phases:

$$W_s(t) = \frac{1}{N_d}\sum_{i=1}^{N_d} W_d^i(t) - \frac{1}{V_f}\int_{V_f} w\, dv, \tag{5}$$

where $W_d^i(t)$ denotes the instantaneous velocity of the *ith* drop and thus the first term on the right-hand side of Eq. (5) represents the instantaneous average velocity of the swarm of drops $W_d(t)$. The second term on the right-hand side of Eq. (5) stands for the volume-averaged velocity in the stratified fluid, where V_f denotes the volume of the continuous phase. The statistically steady-state rise velocity of the swarm W_s is thus obtained by considering the time average of $W_s(t)$:

$$W_s = \frac{1}{t_f - t_i}\int_{t_s}^{t_f} W_s(t)\, dt, \tag{6}$$

where a time period $[t_i, t_f]$ is chosen in such a way to exclude initial transient effects on the average rise velocity. Similarly, the instantaneous and time-averaged velocity fluctuations can be calculated, respectively, as follows

$$W'(t) = \sqrt{\frac{1}{N_d}\sum_{i=1}^{N_d}\left(W_d^i(t) - W_s(t)\right)^2}, \tag{7}$$

and

$$W' = \sqrt{\frac{1}{t_f - t_i}\int_{t_s}^{t_f} W'^2(t)\, dt}. \tag{8}$$

The Reynolds number based on the velocity fluctuation is defined as $Re_{W'} = W'd/\nu_f$.

In this study, we focus on the effects of stratification of the surrounding fluid, deformability of the drop and volume fraction of the swarm on the rising dynamics of drops in a linearly stratified fluid. The Froude number, Eötvos number and volume fraction are varied independently to isolate the above effects, respectively. Unless otherwise stated, we use $Ar = 1100$ corresponding to $Re_W \approx 15 - 25$ in a homogeneous fluid depending on the value of volume fraction and deformability. To model a temperature-stratified fluid, $Pr = 7$ is used in all simulations. Table 1 lists the relevant dimensionless parameters and their range used in the present study. Please note that the range of Froude numbers used in the present study corresponds to density stratifications that are much larger than what is commonly found in oceans ($N \approx 0.01–0.1\ s^{-1}$).