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Inclined two-layered stratified channel flows: Long wave stability analysis of multiple solution regions

R. Kushnir, V. Segal, A. Ullmann, N. Brauner *

School of Mechanical Engineering, Tel Aviv University, Tel Aviv 69978, Israel

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ABSTRACT

In the present work, the linear stability of two-layered stratified channel flows to long wave disturbances is studied. In particular, the study addresses the stability of laminar inclined counter-current and concurrent flows in the regions of multiple solutions for the holdup and pressure drop. The analysis is carried out by solving the Orr-Sommerfeld equations for two-plate geometry, through a formal power series in the wave number. The results are summarized in the form of stability boundaries on flow rate maps, which enable a systematic study of the effect of the system physical parameters on the stratified-smooth/wavy transition in gas–liquid and liquid–liquid systems. It is demonstrated that for counter-current flow there is a region of low flow rates where the two solutions for the holdup are stable. Likewise, the results of concurrent gas–liquid upward flows reveal a region where all three solutions are stable. Moreover, it was found that the middle solution is always stable within the entire 3-s domain. Additionally, the analysis of the wave induced stresses in the axial direction reveals that the terms in phase with the wave slope should be considered in long wave stability analyses of stratified flows.

1. Introduction

Stratified flow is a basic flow pattern in horizontal and inclined gas–liquid and liquid–liquid systems in a gravity field, where a continuous layer of a light phase flow on top of a heavier phase. Stratified two-phase flow regime is frequently encountered in various important chemical and industrial processes. For certain operating conditions however, interfacial instabilities can arise and may produce undesired effects. Therefore, the exploration of the interface stability dependence on the operational parameters is of practical importance.

Exact solutions for steady laminar stratified flow in inclined pipes are available in the literature (e.g. Ullmann et al., 2004). However, an exact formulation of transient flow in pipe geometry is too complicated to conduct a rigorous stability analysis, and the stability boundaries are commonly predicted based on the simplified 1D two-fluid models. On the other hand, exact analysis of the flow in the simpler geometry of two plates can be conveniently carried out to provide insights into the mechanisms involved in the destabilization of stratified flows. Indeed, since the classical work of Yih (1967), the stability of stratified flow in the two-plate geometry has been extensively studied in the literature (e.g. Hooper and Boyd, 1983; Yiantsios and Higgins, 1988; Charru and Fabre, 1994; Kuru et al., 1995). Although, a considerable amount of studies addressed horizontal flow (where the gravity driven multiple solutions are absent), only few referred also to inclined flows (Tilley et al., 1994; Boomkamp and Miesen, 1996; Amaouche et al., 2007; Vempati et al., 2010). In those few studies however, the issue of multiple solutions was not considered.

Exact solutions for steady flow with a smooth interface indicates that in the counter-current region there exist always two possible solutions, whereas in the concurrent up-flow and down-flow a triple solution is obtained in a limited range of the flow parameters (Ullmann et al., 2003a,b, 2004). The introduction of multiple solution regions on flow pattern maps of various two-phase systems shows the practical significance of multiple solutions, and that their boundaries may be associated with flow pattern transition. The feasibility of multiple holdups was also verified experimentally (Ullmann et al., 2003a,b). However, stability analysis is required to determine the range of parameters where the assumption of a smooth interface is valid. Such an analysis may also rule out the feasibility of part of the solutions in the multiple solution regions.

The stability of the flow with respect to long wave disturbances is of particular interest since long wave is an inherent approximation of two-fluid models. In the literature of gas–liquid and liquid–liquid in channels, the instability of the stratified flow pattern is

* Corresponding author. Tel.: +972 3 640 8127; fax: +972 3 640 7334.
E-mail address: brauner@eng.tau.ac.il (N. Brauner).

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usually associated with to the Kelvin–Helmholtz (K–H) mechanism, and the stability analysis is carried out based on the transient two-fluid model equations. The K–H mechanism attributes the growth of interfacial disturbances to the streams inertia forces, which give rise to wave induced pressure fluctuation in phase with the wave height. However, it has been long recognized that the K–H mechanism is not the one responsible for wind-wave generation. The literature on wind waves attributes the energy transfer from the wind to the waves to the Miles-Phillips theory (e.g. Miles, 1962) and to the sheltering mechanism (Jeffreys, 1925), where wave induced pressure fluctuation in phase with the wave slope are responsible for the wind drag and momentum transfer to the waves. According to Miles-Phillips theory, pressure fluctuation in phase with the wave slope results from wind-wave interactions within the critical layer above the water surface, where the air velocity is lower than the wave speed. Deformation of the air flow within the critical layer results in a low pressure at the leeward side and high pressure on the windward side of the wave. According to the Jeffery's sheltering mechanism, the low pressure at the leeward side is attributed to separation of the air flow over the crest, which may occur if the air flow is faster than the wave. However, since during the initial stage of the wave growth, the wind at the wave surface is slower than the wave, the sheltering mechanism is considered to be relevant only at some mature stage of the wave growth, when the local slope becomes larger than a critical value (e.g., Banner and Melville, 1976; Kawai, 1982; Kharif et al., 2008).

In the general case of two-phase flow in channels, it is not apparent whether the wave velocity is faster or slower than the interfacial velocity. Moreover, it is not obvious which of the phases is the faster one and thus dominates the interfacial interactions that lead to instability and wave growth. Nevertheless, it is possible that the interaction between the wave and the flow fields in the two layers will give rise to wave induced stresses in phase with the wave slope. Exploring this possibility in the limit of the long wave approximation is of significance for further analysis and understanding to what extent the above wind-wave generating mechanisms are of importance also in destabilizing the interface between two laminar layers and in the framework of two-fluid models.

This study was undertaken with the purpose of exploring the linear stability of two-layered stratified channel flows to long wave disturbances at operational condition associate with multiple solutions. The analysis was carried out for the general case of counter-current and concurrent flows, by solving the well-known Orr-Sommerfeld equations for the two-plate geometry, through a formal power series in the wave number. It provided a closed form solution for the eigenvalues and eigenfunctions. The results are summarized in the form of stability maps showing the stable and unstable ranges of the various system parameters. The solution is also utilized for deriving the expressions for calculations of the wave induced tangential and normal stresses.

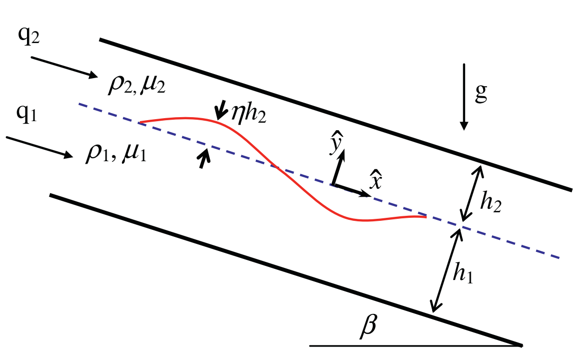

Fig. 1. Schematic description of two-layer flow configuration in an inclined channel.

2. Formulation of the problem

Consider the flow of two immiscible, incompressible fluids, labeled $j = 1, 2$, flowing in an inclined channel ($0 \leqslant \beta \leqslant \pi/2$) as shown in Fig. 1. The flow, assumed isothermal and two dimensional, is driven by an imposed pressure gradient and a component of gravity in the $\hat{x}$ direction. For the indicated coordinate system, the dimensionless continuity and momentum equations governing the flow in each phase are:

$$\frac{\partial u_j}{\partial x} + \frac{\partial v_j}{\partial y} = 0 \tag{1a}$$

$$\frac{\partial u_j}{\partial t} + u_j\frac{\partial u_j}{\partial x} + v_j\frac{\partial u_j}{\partial y} = -\frac{\rho_1}{r\rho_j}\frac{\partial p_j}{\partial x} + \frac{1}{\mathrm{Re}_2}\frac{\nu_j}{\nu_2}\left(\frac{\partial^2 u_j}{\partial x^2} + \frac{\partial^2 u_j}{\partial y^2}\right) + \frac{\sin\beta}{\mathrm{Fr}_2} \tag{1b}$$

$$\frac{\partial v_j}{\partial t} + u_j\frac{\partial v_j}{\partial x} + v_j\frac{\partial v_j}{\partial y} = -\frac{\rho_1}{r\rho_j}\frac{\partial p_j}{\partial y} + \frac{1}{\mathrm{Re}_2}\frac{\nu_j}{\nu_2}\left(\frac{\partial^2 v_j}{\partial x^2} + \frac{\partial^2 v_j}{\partial y^2}\right) - \frac{\cos\beta}{\mathrm{Fr}_2} \tag{1c}$$

These equations are subject to the no-slip impermeable wall boundary conditions

$$y = 1, \quad u_2 = v_2 = 0 \tag{2a}$$

$$y = -n, \quad u_1 = v_1 = 0 \tag{2b}$$

and to the following boundary conditions at the interface, $y = \eta(x, t)$

$$u_1 = u_2, \quad v_1 = v_2 \tag{2c}$$

$$v_j = \frac{\partial \eta}{\partial t} + u_j\frac{\partial \eta}{\partial x} \tag{2d}$$

$$\left[\frac{m\mu}{\mu_1}\left\{\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right)\left(1 - \left(\frac{\partial \eta}{\partial x}\right)^2\right) - 4\frac{\partial u}{\partial x}\frac{\partial \eta}{\partial x}\right\}\right] = 0 \tag{2e}$$

$$\left[p + \frac{m\mu}{\mu_1}\left(\frac{Re_2^{-1}}{1-(\partial\eta/\partial x)^2}\right)\left\{\frac{\partial u}{\partial x}\left(1 - \left(\frac{\partial \eta}{\partial x}\right)^2\right) + \left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right)\frac{\partial \eta}{\partial x}\right\}\right] = \frac{\mathrm{We}_2^{-1}(\partial^2\eta/\partial x^2)}{\left(1+(\partial\eta/\partial x)^2\right)^{3/2}} \tag{2f}$$

Eqs. (2c)–(2f) represent kinematic and dynamic conditions (see Joseph and Renardy, 1993 for more details), where $[f]$ denotes the jump f_2–f_1 across the interface in any quantity f. The dimensionless variables and parameters are defined as follows:

$$(u_j, v_j) = (\hat{u}_j, \hat{v}_j)/u_i, \quad (x, y) = (\hat{x}, \hat{y})/h_2, \quad t = \hat{t}u_i/h_2, \quad p_j = \hat{p}_j/\rho_2 u_i^2$$
$$m = \frac{\mu_1}{\mu_2}, \quad r = \frac{\rho_1}{\rho_2}, \quad n = \frac{h_1}{h_2}, \quad \mathrm{Re}_2 = \frac{u_i h_2}{\nu_2}, \quad \mathrm{Fr}_2 = \frac{u_i^2}{g h_2}, \quad \mathrm{We}_2 = \frac{\rho_2 h_2 u_i^2}{\sigma} \tag{3}$$

where $\hat{u}_j$, $\hat{v}_j$, and $\hat{p}_j$ are the velocities components and pressure of fluid j, μ_j, ν_j and ρ_j are the corresponding dynamic viscosity, kinematic viscosity and density, and $\hat{t}$, g and σ denote the time, gravitational acceleration, and interfacial surface tension, respectively. As seen, the length, velocity, time and pressure scales are h_2, u_i, h_2/u_i