Minerals Engineering 126 (2018) 82–88

Contents lists available at ScienceDirect

Minerals Engineering

journal homepage: www.elsevier.com/locate/mineng

Statistical reliability of the liberation distribution of ore particles with respect to number of particle measurements

Takao Ueda*, Tatsuya Oki, Shigeki Koyanaka

Environmental Management Research Institute, Department of Energy and Environment, National Institute of Advanced Industrial Science and Technology (AIST), 16-1 Onogawa, Tsukuba, Ibaraki 305-8569, Japan

ARTICLE INFO

Keywords:
Statistical reliability
Mineral liberation
Liberation distribution
Numerical simulation

ABSTRACT

Automated mineral liberation analyzers, which have recently seen widespread use, enable us to obtain liberation distribution information based on sectional measurements of a great number of ore particles. A statistically based method was here developed, which correlates the confidence interval of each bin of the liberation distribution with the requisite minimum number of sectional particle measurements. Its primary feature is the estimation, by statistical means, of the required number of particle measurements from probabilities corresponding to the frequencies of the respective bins. The noteworthy feature of the method is that it determines the required number of particle measurements with respect to the specific needs of statistical reliability, such that the confidence interval for volumes over 90% content of the mineral of interest is less than 0.01, and so on. A series of numerical validation simulations was conducted, in which 100,000 binary particles were modeled and sampling of the estimated required number of particles was conducted 1000 times. Three types of liberation distributions were investigated, based on the respective assessment of two-dimensional (2D) sectional area, 2D particle number, and three-dimensional particle number. If the measurement reliability of a given sampling satisfied the designated confidence level, it was considered to meet the required quantitative reliability for liberation distribution. However, the measurement reliability for 2D areal liberation distribution failed to satisfy the requirement, and the reason for this, as well as a possible solution, are also discussed.

1. Introduction

Accurate assessment of the liberation state for ground mineral particles is important for achieving effective mineral processing. The liberation state is generally represented by the *degree of liberation*, which is the fractional amount of liberated particles containing the mineral of interest, compared to the total amount of the mineral in the particle system. Another pertinent factor is the *liberation distribution*, the cumulative distribution of the amount of particles with respect to their content of the mineral phase of interest (x). Since the liberation distribution includes the concept of the degree of liberation (i.e., both ends of the liberation distribution ($x = 0$and1) are relevant to the respective degrees of liberation), the liberation distribution was the primary focus of this study. The liberation distribution is originally a three-dimensional (3D) factor. However, since the 3D liberation state is difficult to determine practically, it is generally treated as a two-dimensional (2D) factor. As a 2D factor, the respective cumulative distributions of the area and number of particle sections are separately employed, in accordance with the requirements. The liberation distribution is originally a continuum function of x, but in practice is often treated as involving twelve bins. When the bins are designated as i, $i = 1{-}12$ corresponds to $x = 0$, $0{-}0.1$, $\cdots, 0.9{-}1$, 1, respectively.

The 2D liberation distribution is obtained by an analysis of polished sections of resin-mounted mineral particle samples. Scanning electron microscope and energy dispersive X-ray analysis based on automated analyzers (e.g., Mineral Liberation Analyzer (Fandrich et al., 2007), Quantitative Evaluation of Minerals by Scanning Electron Microscopy (Gottlieb et al., 2000), and TESCAN Integrated Mineral Analyzer (TESCAN, 2012), both recently in widespread use, are powerful tools for rapid measurement of the 2D liberation distribution. As a next step, a method must be developed to correlate the confidence interval of the liberation distribution with the number of particle measurements, and thereby enable effective measurement of the requisite minimum number of particle sections with respect to the required liberation distribution confidence interval.

The following studies are concerned with the statistical reliability of the liberation state of ore particles.

Leigh et al. (1993) derived a deviation model for liberation

* Corresponding author.
E-mail address: t-ueda@aist.go.jp (T. Ueda).

https://doi.org/10.1016/j.mineng.2018.06.024
Received 27 March 2018; Received in revised form 16 May 2018; Accepted 21 June 2018
0892-6875/

distribution, and compared it with the bootstrapping technique (Efron, 1979). Bootstrapping is a method to estimate the statistic of parent population using empirical distributions obtained by resampling data. They selected 1000 samples using the bootstrapping technique, measured their respective liberation state, and estimated the deviation using the arcsine transformation. The model showed good agreement with the bootstrapping test.

Evans and Napier-Munn (2013) proposed a method to estimate the relationship between the number of particle section measurements and the variation coefficient. They selected 25 to 750 particle sections and measured their liberation state using the bootstrapping technique. The variation coefficient was calculated with 1000 repetitions, and its relation to the number of particle section measurements was expressed by the equation: $y = ax^b$. Using this equation, the variation coefficient for samples with larger particle numbers was successfully predicted.

Whereas the abovementioned studies dealt with the deviation of the whole liberation distribution curve, Mariano and Evans (Mariano and Evans, 2015) focused on the confidence interval for each bin of liberation distribution. They measured the confidence interval for each bin of copper ore samples, using MLA measurement and the bootstrapping technique.

Ueda et al. (2016) derived a relationship between the confidence interval for the degree of liberation and the number of particle section measurements, by statistical means, and validated this through a series of numerical simulations.

As an extension of the abovementioned studies, a specific method was developed in the present study, which correlates the confidence interval for each bin of liberation distribution with the required number of particle section measurements. This method determines a requisite minimum number of particle section measurements with respect to the specific condition of the confidence interval for each bin (e.g., the confidence interval must be less than 0.01 over the range $x \geq 0.9$). In addition, a method for practical implementation is proposed. Finally, a series of numerical simulations was conducted to validate the proposed method.

2. Methodology

This section describes the proposed statistical reliability method for liberation distribution, and a method for practical implementation. Two-types of 2D sectional assessment (areal-based and number-based), and 3D number assessment will be considered. Number-based liberation distribution has traditionally been used for manual microscopy measurement, whilst the recently popularized automated liberation analyzer makes it possible to analyze the areal-based liberation distribution. In this study, these two types of the 2D liberation distributions will be considered. Today, mineral liberation assessment is typically conducted based on 2D sections. However, if 3D particle information can be obtained, such as by X-ray computed tomography (Lin and Miller, 1996; Miller et al., 2009; Videla et al., 2007; Miller et al., 2003; Gay and Morrison, 2006), serial particle sectioning (Lätti and Adair, 2001; Schneider et al., 1991; Miller and Lin, 1988), or stereological correction (Gay and Morrison, 2006; Miller and Lin, 1988; King and Schneider, 1998; Leigh et al., 1996; Gay, 1994; Ueda et al., 2016, 2017), the below procedure will be equally applicable to the 3D liberation assessment.

2.1. Statistical reliability method for liberation distribution

Binary particle systems in which the particles are composed of phases A and B are considered. Of course, actual ore particles are composed of multiple mineral phases, but if the mineral phases are divided into the mineral of interest and others, the ore particles can be treated as binary particles. In this manner, investigation of binary particles is applicable to actual ore particles without loss of generality. The content of phase A in the particle section is redefined as x ranging from zero to one. Hence, the particle section with $x = 0$ is apparently liberated with phase B, and that with $x = 1$ is apparently liberated with phase A. As noted in the Introduction, the bins are designated as i, and $i = 1–12$ corresponds to $x = 0,\ 0–0.1,\ \cdots, 0.9–1,\ 1$, respectively.

The probability of a given particle section entering the i-th bin for the parent population (P_i) is

$$P_i = \begin{cases} S_i/S_{all} & \text{in 2D(area)} \\ Q_i/Q_{all} & \text{in 2D(number)}, \\ T_{i/T_{all}} & \text{in 3D(number)} \end{cases} \tag{1}$$

where S_i, Q_i, and T_i denote the sums of sectional areas, number of particle sections, and number of particles in the i-th bin, respectively, and S_{all}, Q_{all}, and T_{all} denote the total sectional areas, total number of particle sections, and total number of particles, respectively. Similarly, the probability of a given particle section entering the i-th bin for the sample ($\widehat{P_i}$) is

$$\widehat{P_i} = \begin{cases} \widehat{S_i}/\widehat{S}_{all} & \text{in 2D(area)} \\ \widehat{Q}_i/\widehat{Q}_{all}\widehat{Q}_i & \text{in 2D(number)}, \\ \widehat{T}_i/\widehat{T}_{all} & \text{in 3D(number)} \end{cases} \tag{2}$$

where the hat represents the parameters of the limited number of samples.

When the confidence interval for the i-th bin is ξ_i (i.e, when $\widehat{P_i}$ is expected to be in the range of $P_i \pm \xi_i$), the required particle section number in the i-th bin (N_i) is determined as follows:

$$N_i = \left(\frac{K_P}{\xi_i}\right)^2 \widehat{P_i}\,(1-\widehat{P_i}), \tag{3}$$

where K_P is a value that satisfies the following equation, giving $K_P = 1.96$ for a required reliability (R_r) of 95% from the table of normal distribution:

$$\text{Prob}\{K_P \leq u\} = \frac{1-0.95}{2}, \tag{4}$$

where u is a variant that obeys the standard normal distribution.With arbitrarily specified ξ_is, various statistical reliability conditions for the liberation distribution can be assigned in accordance with the requirements. Table 1 presents three cases as examples.

The required number of particle section measurements (N_r) is calculated as follows:

$$N_r = \max_{i=1-12} N_i. \tag{5}$$

2.2. Practical implementation

The proposed method can be used in two different ways, during and after the measurements, as follows.

In the first case, the required number of particle section measurements can be determined during the measurement, which leads to effective measurement without wasted cost and time. More concretely,

Table 1
Examples of statistic reliability requirements for bins.

Case	Statistical reliability requirements	Specified ξ_i
1	The confidence interval (ξ_i) for all bins should be less than p.	$\xi_i = p$ where $i = 1–12$
2	ξ_i for bins with $x \geq 0.8$, $(i \geq 10)$ should be less than p, because the range with high phase A content in the liberation distribution is concerned.	$\xi_i = \begin{cases} p & \text{where } i = 10–12 \\ \infty & \text{where } i = 1–9 \end{cases}$
3	ξ_i for bins with $x = 0\text{and}1$, $(i = 1\text{and}12)$ should be less than p, because the degrees of liberation of phases A and B are concerned.	$\xi_i = \begin{cases} p & \text{where } i = 1 \text{ and } 12 \\ \infty & \text{where } i = 2–11 \end{cases}$