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Numerical simulations of compressible flows using multi-fluid models

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ABSTRACT

Numerical simulations of two-fluid flow models based on the full Navier–Stokes equations are presented. The models include six and seven partial differential equations, namely, six- and seven-equation models. The seven-equation model consists of a non-conservative equation for volume fraction evolution of one of the fluids and two sets of balance equations. Each set describes the motion of the corresponding fluid, which has its own pressure, velocity, and temperature. The closure is achieved by two stiffened gas equations of state. Instantaneous relaxation towards equilibrium is achieved by velocity and pressure relaxation terms. The six-equation model is deduced from the seven-equation model by assuming an infinite rate of velocity relaxation. In this model, a single velocity is used for both fluids. The numerical solutions are obtained by applying the Strang splitting technique. The numerical solutions are examined in a set of one, two, and three dimensions for both the six- and seven-equation models. The results indicate very good agreement with the experimental results. There is an insignificant difference between the results of the two models, but the six-equation model is much more economical compared to the seven-equation model.

Introduction

Multi-fluid flow is known as the simultaneous flow of more than one fluid phase. The fluids may be at different phases of either the same or different substances. Examples include liquid and its vapor flows in evaporators, condensers, and over torpedoes; gas–liquid flows in an oil production pipeline system; liquid–solid flows in blood and rivers; and gas–liquid–solid flows in chemical reactors. Typically, the flow is under large disparate conditions of pressure, density, and flow speed. The density ratios are typically in the order of 10^3–10^5 and with a sound speed ratio of 10^2–10^3. These types of flows are mainly responsible for difficulties in modeling and numerical methods.

There are two general approaches to describe multi-fluid flows: the Euler–Lagrange approach and the Euler–Euler approach. In the Euler–Lagrange approach (Giannadakis et al., 2008), one fluid phase is the continuum, while the others are considered dispersed phases such as particles, bubbles, or droplets. In this approach, the balance equations are solved for the continuous fluid, and the dispersed phase trajectories are tracked by integrating the equations of motion for each dispersed phase. Since this approach captures the flow features at the level of a single dispersed phase, it is commonly quite expensive. In the Euler–Euler approach (Bear and Nunziato, 1986; Kapila et al., 2001; Layes and Métayer, 2007; Lindau et al., 2006; Merkle et al., 2006; Murrone and Guillard, 2005; Owis and Nayfeh, 2003; Saurel and Abgrall, 1999; Saurel et al., 2009; Venkateswarana et al., 2002), all fluids are considered continua. The fluid phases are treated separately using multiple sets of balance equations. Each set describes the motion of the corresponding fluid, which has its own pressure, velocity, and temperature. In addition to the balance equations, a transport equation for the volume fraction evolution of one of the fluids is also solved to track the different fluids. This approach is usually called the multi-fluid seven-equation model, or the parent model (Saurel and Abgrall, 1999). It allows the most general description of multi-fluid flows. However, it is considerably expensive because it deals with the solution of a large system of equations, that is, 12 equations for a two-dimensional (2D) two-fluid case and 14 equations for a three-dimensional (3D) case. A number of "reduced" compressible models based on the two-fluid seven-equation model have been developed in the literature. The dependent variables can be generally categorized into three groups: six-equation models (either having two pressures with a single velocity and two temperatures or a single pressure, two velocities, and two temperatures) (Saurel et al., 2009), five-equation models (using a single pressure, a single velocity, and two temperatures) (Kapila et al., 2001; Murrone and Guillard, 2005), and four-equation models (homogeneous equilibrium models [HEMs]) (Lindau et al., 2006; Owis and Nayfeh, 2003). Among these groups, the HEMs

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are the simplest and are obtained from the patent model with the assumptions that all fluids share the same pressure, velocity, and temperature. In this case, the mass, momentum, and energy balance equations are sufficient to describe the flows. Although the assumptions of the HEMs clearly indicate some limitations, they can still be quite adequate for certain flow conditions (Ha et al., 2012a, b; Merkle et al., 2006).

This study aims at developing and examining a numerical solution procedure for the six- and seven-equation models. The paper is presented in five sections. The introduction is followed by the presentation of the two-fluid models. The third section presents the numerical solution procedures. The fourth section presents the numerical results and discussions. A summary of the completed work and the concluding remarks of the study are given at the end of the paper.

Two-fluid models

By neglecting all turbulent terms, the viscosity, the surface tension, the surface energy, and the heat terms, everywhere except at the interfaces, the extension of the two-fluid seven-equation model (Saurel and Abgrall, 1999) to the multidimensional problem is given as follows:

$$\frac{\partial U}{\partial t}+\frac{\partial E}{\partial x}+\frac{\partial F}{\partial y}+\frac{\partial G}{\partial z}=H_x\frac{\partial \alpha_1}{\partial x}+H_y\frac{\partial \alpha_1}{\partial y}+H_z\frac{\partial \alpha_1}{\partial z}+V^R(U)+P^R(U)+S^R(U) \tag{1}$$

where

$$U=\begin{pmatrix}\alpha_1,\alpha_1\rho_1,\alpha_1\rho_1u_1,\alpha_1\rho_1v_1,\alpha_1\rho_1w_1,\alpha_1\rho_1E_1\\ \alpha_2\rho_2,\alpha_2\rho_2u_2,\alpha_2\rho_2v_2,\alpha_2\rho_2w_2,\alpha_2\rho_2E_2\end{pmatrix}^T \tag{2a}$$

$$E=\begin{pmatrix}0,\alpha_1\rho_1u_1,\alpha_1\rho_1u_1^2+\alpha_1p_1,\alpha_1\rho_1u_1v_1,\alpha_1\rho_1u_1w_1\\ \alpha_1u_1(\rho_1E_1+p_1),\alpha_2\rho_2u_2,\alpha_2\rho_2u_2^2+\alpha_2p_2,\\ \alpha_2\rho_2u_2v_2,\alpha_2\rho_2u_2w_2,\alpha_2u_2(\rho_2E_2+p_2)\end{pmatrix}^T \tag{2b}$$

$$F=\begin{pmatrix}0,\alpha_1\rho_1v_1,\alpha_1\rho_1v_1u_1,\alpha_1\rho_1v_1^2+\alpha_1p_1,\alpha_1\rho_1v_1w_1,\\ \alpha_1v_1(\rho_1E_1+p_1),\alpha_2\rho_2v_2,\alpha_2\rho_2v_2u_2,\\ \alpha_2\rho_2v_2^2+\alpha_2p_2,\alpha_2\rho_2v_2w_2,\alpha_2v_2(\rho_2E_2+p_2)\end{pmatrix}^T \tag{2c}$$

$$G=\begin{pmatrix}0,\alpha_1\rho_1w_1,\alpha_1\rho_1w_1u_1,\alpha_1\rho_1w_1v_1,\alpha_1\rho_1w_1^2+\alpha_1p_1,\\ \alpha_1w_1(\rho_1E_1+p_1),\alpha_2\rho_2w_2,\alpha_2\rho_2w_2u_2\\ \alpha_2\rho_2w_2v_2,\alpha_2\rho_2w_2^2+\alpha_2p_2,\alpha_2w_2(\rho_2E_2+p_2)\end{pmatrix}^T \tag{2d}$$

$$\begin{aligned}H_x&=(-u_I,0,p_I,0,0,p_Iu_I,0,-p_I,0,0,-p_Iu_I)^T\\ H_y&=(-v_I,0,0,p_I,0,p_Iv_I,0,0,-p_I,0,-p_Iv_I)^T\\ H_z&=(-w_I,0,0,0,p_I,p_Iw_I,0,0,0,-p_I,-p_Iw_I)^T\end{aligned} \tag{2e}$$

$$V^R(U)=\begin{pmatrix}0,0,\lambda(u_2-u_1),\lambda(v_2-v_1),\lambda(w_2-w_1),\\ \lambda[(u_2-u_1)u_I+(v_2-v_1)v_I+(w_2-w_1)w_I],\\ 0,-\lambda(u_2-u_1),-\lambda(v_2-v_1),-\lambda(w_2-w_1),\\ -\lambda[(u_2-u_1)u_I+(v_2-v_1)v_I+(w_2-w_1)w_I]\end{pmatrix}^T \tag{2f}$$

$$P^R(U)=\begin{pmatrix}-\mu(p_2-p_1),0,0,0,0,-\mu p_I(p_2-p_1)\\ 0,0,0,0,\mu p_I(p_2-p_1)\end{pmatrix}^T \tag{2g}$$

$$S^R(U)=\begin{pmatrix}0,0,\alpha_1\rho_1g_x,\alpha_1\rho_1g_y,\alpha_1\rho_1g_z,\\ 0,0,\alpha_2\rho_2g_x,\alpha_2\rho_2g_y,\alpha_2\rho_2g_z,0\end{pmatrix}^T \tag{2h}$$

where U is the conservative vector; E, F, and G are the inviscid flux vectors in the x, y, and z directions, respectively; H represents the volume, momentum, and energy exchange between two fluids; V^R represents the velocity relaxation term; P^R represents the pressure relaxation term; μ and λ represent the rates of pressure relaxation and velocity relaxation, respectively; S^R represents the source term; E_k is the total energy for the fluid k ($k=1,\ 2$), $E_k=e_k+(u_k^2+v_k^2+w_k^2)/2$; e_k, α_k, ρ_k, u_k, and p_k are the internal specific energy, volume fraction, density, velocity, and pressure for the fluid k, respectively; and p_I and u_I are the mixture pressure and velocity, respectively.

The mixture density, pressure, and velocity are defined as

$$\begin{aligned}&\rho=\sum_{k=1}^{2}\alpha_k\rho_k,p_I=\sum_{k=1}^{2}\alpha_kp_k,\\ &u_I=\sum_{k=1}^{2}\alpha_k\rho_ku_k\Big/\sum_{k=1}^{2}\alpha_k\rho_k\end{aligned} \tag{3}$$

The compatibility relation for the volume fraction is given as

$$\sum_{k=1}^{2}\alpha_k=1 \tag{4}$$

In the derivation of a reduced model, i.e., the six-equation model, we used the assumption of an infinite rate of velocity relaxation ($\lambda\to\infty$) to define a single velocity ($u_1=u_2=u$, $v_1=v_2=v$, and $w_1=w_2=w$). Subsequently, by summing the two momentum equations and cancelling out the velocity relaxation term in the seven-equation model, the six-equation model can be obtained as follows:

$$\frac{\partial U}{\partial t}+\frac{\partial E}{\partial x}+\frac{\partial F}{\partial y}+\frac{\partial G}{\partial z}=H_x\frac{\partial \alpha_1}{\partial x}+H_y\frac{\partial \alpha_1}{\partial y}+H_z\frac{\partial \alpha_1}{\partial z}+P^R(U)+S^R(U) \tag{5}$$

where

$$U=(\alpha_1,\alpha_1\rho_1,\rho u,\rho v,\rho w,\alpha_1\rho_1E_1,\alpha_2\rho_2,\alpha_2\rho_2E_2)^T \tag{6a}$$

$$E=\begin{pmatrix}0,\alpha_1\rho_1u,\rho u^2+p_I,\rho uv,\rho uw,\alpha_1u(\rho_1E_1+p_1),\\ \alpha_2\rho_2u,\alpha_2u(\rho_2E_2+p_2)\end{pmatrix}^T \tag{6b}$$

$$F=\begin{pmatrix}0,\alpha_1\rho_1v,\rho vu,\rho v^2+p_I,\rho vw,\alpha_1v(\rho_1E_1+p_1),\\ \alpha_2\rho_2v,\alpha_2v(\rho_2E_2+p_2)\end{pmatrix}^T \tag{6c}$$

$$G=\begin{pmatrix}0,\alpha_1\rho_1w,\rho wu,\rho wv,\rho w^2+p_I,\alpha_1w(\rho_1E_1+p_1),\\ \alpha_2\rho_2w,\alpha_2w(\rho_2E_2+p_2)\end{pmatrix}^T \tag{6d}$$

$$\begin{aligned}H_x&=(-u,0,0,0,0,p_Iu,0,-p_Iu)^T,\\ H_y&=(-v,0,0,0,0,p_Iv,0,-p_Iv)^T,\\ H_z&=(-w,0,0,0,0,p_Iw,0,-p_Iw)^T\end{aligned} \tag{6e}$$

$$P^R(U)=\begin{pmatrix}-\mu(p_2-p_1),0,0,0,0,\\ -\mu p_I(p_2-p_1),0,\mu p_I(p_2-p_1)\end{pmatrix}^T \tag{6f}$$

$$S^R(U)=(0,0,\rho g_x,\rho g_y,\rho g_z,0,0,0)^T \tag{6g}$$

For the system closure, the stiffened gas (SG) equation of state (EOS) is used for each pure fluid as

$$p_k=(\gamma_k-1)\rho_ke_k-\gamma_k\pi_k \tag{7}$$

where γ_k and Π_k are the specific heat ratio and reference pressure of fluid k, respectively. These two parameters can be determined by using a calibration method as suggested by Cocchi et al. (1996). For a gas, $\Pi_k=0$ and the SG EOS is identical to the ideal gas EOS.