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On the size dependence of the King stratification index

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ABSTRACT

The study reported in this paper addresses an aspect of the modelling of particle stratification as it applies to the processing of minerals. It focuses on the phenomenological model developed by King and the extent to which the single parameter in that model, i.e. the stratification index, is dependent on the size of particles in the stratifying system. Based on a careful experimental investigation in a batch jig, the study found that, contrary to expectation and current theory, the size dependence of the index is weak or absent over the range of conditions investigated. The implications of the findings for both theory and modelling practice and for Rao's proposed modification of King's model are discussed. The study also presents a number of findings related to experimental procedures for obtaining precise measurements of parameters in stratification models.

1. Introduction

Several mineral separation processes are based on exploiting differences in particle density by engineering a stratification of the particles in a bed in an aqueous environment and then removing the layer that contains the majority of the valuable mineral. To model the stratification that occurs in such processes has proved a challenge. Even in the simplest application of stratification in a mineral separation process, i.e. the batch jig, there are several interacting dynamics at play and each is difficult to model. More complex systems, such as continuous jigging or spirals, involve additional dynamics that present additional difficulties. This paper avoids the issues associated with these other dynamics by focusing only on stratification in a batch jig.

Crespo (2016) has pointed out that it was only in the mid 1980's that non-empirical models emerged that could predict jig performance with some success. In particular he highlighted the phenomenological model by King (King, 1987, 2001; Tavares and King, 1995) and the mechanistic models that have developed since the early 1990's that simulate the movement of particles in a jig bed (for example see Mishra and Mehrotra, 2001; Asakura et al., 2007; Mukherjee and Mishra, 2007; Viduka et al., 2012; Crespo 2016). This paper focuses on the King model.

The model requires only one experimentally determined parameter, the stratification index, to describe the stratification patterns in the bed given the proportion and density of each particle component in that bed. The model has been well validated in a variety of contexts (King, 1987; Tavares and King, 1995; Woollacott et al., 2015) but has a number of limitations. The most serious of these is that it does not account for variations in particle size; it assumes that all particles in a stratifying system have the same size. Given the success of the model in describing stratification in some contexts, it is of considerable interest to establish the degree to which its predictive power extends to systems where there is some variation in particle size.

Rao (2007) has suggested that the effect of particle size on stratification might be modelled by focusing on the size dependence of the stratification index in King's model. On this basis, he suggested a modification to King's model but presented no experimental evidence to validate his proposal. Accordingly, this paper addresses two related issues – the influence of particle size on the predictive power of King's model and the veracity of Rao's modification of that model. The paper addresses these issues by investigating the size dependence of the King stratification index. It begins by reviewing the relevant theory after which the experimental study is presented and its findings are discussed.

2. Theory: the size dependence of King's stratification index

The conceptual breakthrough behind the King model is the idea that the stratification patterns in a particle bed are the result of a dynamic equilibrium between a stratification driving force and an opposing dispersive force. Following Mayer (1964), King assumed that stratification was driven by the reduction in potential energy that occurs when particles of different density stratify. He assumed that normal diffusive processes that tend to flatten out any concentration gradient in a context where mobility of components is possible would oppose the tendency to stratify. The resulting model, Eq. (1), is elegant and requires

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only one experimentally determined parameter, the stratification index **α**.

$$\frac{dC_j(h)}{dh} = -\alpha C_j(h)(\rho_j - \bar{\rho}(h)) \quad (1)$$

In the equation, C_j (h) is the volumetric concentration of particle component j in the differentially thin layer, dh thick, located at a relative height h from the bottom of the bed; $h = H/H_{bed}$ where H is the actual height of the thin layer from the bottom of the bed, and H_{bed} is the overall height of the bed. The jth particle component has a density ρ_j and $\bar{\rho}$ (h) is the mean density of the particles in the layer at h.

According to King (1987, 2001), the stratification index is a composite of a number of factors as shown in Eq. (2).

$$\alpha = gV_{part}H_{bed}\bar{u}/D \quad (2)$$

This equation suggests that the value of the stratification index depends on the depth of the bed (H_{bed}), the volume of individual particles (V_{part}), the gravitational constant g, and two factors that account for the mobility of the particles in the bed due to diffusive processes (the factor D) and to stratification processes (the factor $\bar{u}$). D is the Fickian coefficient of diffusion associated with processes that tend to homogenize any concentration gradients created by stratification processes. The term $\bar{u}$ is the specific particle mobility in the bed that drives stratification processes. It derives from the reduction in the potential energy of the bed (Mayer, 1964) that results when mono-sized particles of different density stratify (King 1987, 2001; Tavares and King, 1995).

The terms $\bar{u}$ and D, taken together, account for the effect of the specific segregation conditions that prevail in a bed as it stratifies. They are expected to be strongly dependent on particle size (King, 2001). However, the value of the compound term$\bar{u}/D$, as well as its size dependence, can only be determined experimentally by finding the value of the stratification index that leads to a best fit of the model to experimental data. Rao, noting this and the size dependence of a third term in Eq. (2), i.e. the volume of the particles, V_{part}, suggested reformulating the equation to capture the size dependence of the stratification index as a whole in terms of the simple power relationship shown in Eq. (3).

$$\alpha(\ell_j) = A\ell_j^b \quad (3)$$

Here ℓ_j is the nominal diameter of the jth particle component, and A and b are parameters whose values must be determined experimentally. While Rao did not have any experimental data to test the veracity of his proposal, he did show that simulations based on his modification were able to generate performance information that was similar to the known general nature of the performance of jig separators.

It should be noted, however, that the modification of the King model that Rao proposed does not constitute a generally applicable model of the effect of both size and density on stratification. This can be demonstrated quite simply by noting that the modified model fails to account for size segregation in all circumstances. For example, it incorrectly predicts that no size segregation will occur if all particles have the same density; the term dCj/dh in Eq. (1) reduces to zero in that context. Nevertheless, the modification is of interest in that it may have utility in extending the applicability of King's model in situations where density differences are the primary factor driving stratification.

3. Background to the experimental study

3.1. Estimation of the stratification index

The value of King's stratification index in a given context is estimated by fitting the model to relevant experimental data which is typically obtained by slicing layers from a stratified particle bed and determining the concentration of the different particle components in each layer. Model predictions for binary, mono-sized particle systems can be obtained by explicit solution of Eq. (1) (see King, 2001, p. 243–249) leading to Eq. (4) for $C_1(slice)$, the concentrati on of component 1 in the slice from $h = h_{bottom}$ to $h = h_{top}$.

$$C_1(slice) = -\frac{1}{\beta(h_{top}-h_{bottom})}\ln\left(\frac{1+K\ exp(-\beta h_{top})}{1+K\ exp(-\beta h_{bottom})}\right) \text{ (for the slice from } h = h_{bottom} \text{ to } h = h_{top}) \quad (4)$$

In this equation, $\beta = \alpha(\rho_1 - \rho_2)$ and ρ_1 and ρ_2 are respectively the densities of the denser and less dense components. K is defined in Eq. (5) and is derived from the boundary condition for the known concentrations of the two components in the bed as a whole, i.e. for $C_1 = C_1^{bed}$ and $C_2 = C_2^{bed}$ when $h_{bottom} = 0$ and $h_{top} = 1$

$$K = \frac{1-exp(-\beta C_1^{bed})}{exp(\beta C_2^{bed})-1}exp(\beta) \quad (5)$$

3.2. Discrimination in the estimation of the stratification index

Fig. 1 illustrates the sensitivity of the concentration profiles to the value of the stratification index for binary systems where the density difference of particles is around 300 kg/m^3, i.e. the conditions tested in the study. For this context, the figure highlights two features that had a strong bearing on the experimental procedure adopted in the study. First, when the stratification index varies in the range 0.090–0.120 m^3/kg, as it did in the study, the greatest discrimination between profiles associated with different stratification indices occurs in the regions in Fig. 1 labelled the 'shoulders' of the profile. The implication of this is that, in order to maximize discrimination when estimating the value of the index, the bed must be sliced in such a way as to have as many data points in the region of the 'shoulders' as possible. If the data points occur only above, below and in-between these 'shoulders', the concentration profiles for stratifications with indices from about 0.090 to 0.120 m^3/kg are likely to be experimentally indistinguishable.

The second feature to note is in Fig. 1B. It shows the relative thickness of slices 15 and 10 mm thick removed from an 80 mm deep bed, i.e. conditions typical of the experimental setup used in the study. What is apparent from the figure is that a profile generated from a single test by slicing the bed in this context will provide at best two or perhaps three data points in the two shoulder regions. Therefore, in order to discriminate appropriately when estimating the value of the index for a specific profile in this context, a series of individual tests are needed so that multiple data points in both 'shoulder' regions are obtained. In addition, the heights at which the bed is sliced in these tests needs to be slightly different and carefully selected to enhance the definition of the concentration profiles in those regions.

3.3. The relative height of a layer, *h*

The King model describes the concentration profiles in the bed in terms of the relative height h of a layer from the bottom of the bed. Therefore, to maximize the accuracy of the parameter estimation requires accurate measurement of H and H_{bed} (h being equal to H/H_{bed}). In practice, the measurement of the latter was difficult because the top of a particle bed that has been subjected to pulsating water movement invariably is not flat, is loosely packed, and frequently one or several particles stand proud from their neighbours. This meant that there was considerable uncertainty in the measurement of H_{bed}. Depending on how the top of the bed was flattened and levelled (and, in the process, compacted somewhat), repeated measurements of H_{bed} varied by as much as 4 mm, i.e. by up to 5% in an 80 mm deep bed. To avoid the resulting uncertainty in the value of h, the relative height of a layer was determined by reference to the volumetric split of the particles achieved when the bed was sliced, i.e. as h_v defined in Eq. (6), where A is the