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A review on determination of particle–bubble encounter using analytical, experimental and numerical methods

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ABSTRACT

This paper provides a comprehensive critical review of available studies on analytical and numerical modeling including computational fluid dynamics (CFD), as well as experimental approaches to determine the particle–bubble interactions in flotation cells. The effects of some significant factors such as particle density, bubble size and velocity, and cell turbulence on the particle–bubble encounter are investigated in detail. This review indicates that interception collision models established based on stream functions are not applicable as they ignore the turbulence effect. The streamlines are not stationary in turbulent conditions and constantly change throughout time and space. Furthermore, the analytical models are restricted because of poor estimation of collision angle, Stokes numbers, effect of particle density and disregarding microhydrodynamic forces and turbulence effects. Unlike analytical modeling, numerical modeling is a very powerful technique for evaluating particle–bubble encounter interactions. The role of particle density and turbulence in particle–bubble encounter can be best identified by numerical methods. However, there is lack of experimental data to verify these models. Therefore, more specific and direct measurement techniques are required to develop accurate estimation of particle–bubble encounter probabilities. This review finally highlights the gaps in the evaluation of particle–bubble encounter efficiency and recommends further works to investigate relationships between hydrodynamic properties, particle–bubble characterizations, flotation kinetic rates and particle–bubble encounter interactions.

1. Introduction

Flotation processes can be studied from macroscopic and microscopic points of view. The macroscopic aspect focuses on chemical properties (water quality, pH, reagent type and dosage), equipment factors (gas flowrate, turbulence, hydrodynamics of the cell and power input) and operational properties (particle size, pulp density, circuit design, feed rate and mineralogy of ore). However, microscopic perspective of particle–bubble interactions is classified into collision, attachment and stability sub–processes. Since measurements of macroscopic parameters are much easier than microscopic characterizations, many studies have been devoted to them. Nevertheless, over the last decades, understanding of particle–bubble interaction mechanisms have been remarkably enhanced owing to identifying their significance role in flotation processes and also convenient access to modern utilities such as Atomic Force Microscopy (AFM), high–speed cameras and new developments in numerical methods (Zhang and Finch, 2001; Duan et al., 2003; Nguyen et al., 2006; Sarrot et al., 2007; Assemi et al., 2008; Liu and Schwarz, 2009a,b; Shahbazi et al., 2010; Firouzi et al., 2011; Kouachi et al., 2017).

Particle–bubble encounter (so–called collision) is considered as the most effective sub–process of particle–bubble interaction because of its significant direct impact on flotation rate constant and flotation recovery. Studies on determining particle–bubble interactions can be categorized into three main methods, i.e. numerical modeling, analytical modeling and experimental measurements of these interactions. Direct experimental observations of the particle–bubble encounter require very sophisticated equipment because it is difficult to isolate particle–bubble encounter from other sub–processes (attachment and detachment) during actual flotation separation. Indeed, the processes are affected by many interrelated factors. For this reason, experimental studies have mostly focused on a single bubble rising in a very simplified fluid flow or measurements of collection efficiency (E_{coll}) (so–called capture efficiency, E_{cap}) rather than particle–bubble encounter interactions. However, results reported by Huang et al. (2011) disclosed that the error of E_{cap} in terms of measuring the accuracy for several experiments was more than 50%. Recently, several researchers attempted to measure particle–bubble encounter efficiency (E_c) using

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practical approaches by means of high–speed cameras (Dai, 1998; Nguyen and Evans, 2002; Shahbazi et al., 2010; Basarova et al., 2010a,b; Ireland and Jameson, 2014; Brabcova et al., 2015). In fact, their studies suffered from using only glass beads and/or quartz particles or experiments conducted in quiescent systems in the absence of turbulent effect. Since these experiments were implemented on just quartz particles, particle density dependence of inertial effect was totally ignored. Also, as the experiments did not take into account the liquid flow and turbulence of the cell, the collision of particles with bubbles was solely based on the settling velocity of particles. Moreover, as the bubbles were held at the bottom, the liquid motion near the surface of the bubble was modified. Measurements of the critical radius are also very difficult as it requires the particle trajectory remaining in the symmetry plane of the bubble. Another problematic case in experimental studies is cleaning the surface of glass beads. Most of the studies used acid and alkali treatments methods (Dai, 1998; Pyke et al., 2003; Guven et al., 2015; Vaziri Hassas et al., 2016). However, it is now clearly demonstrated that the treatment with acid or alkaline agents increases the negative charge on the glass surface. Positive disjoining pressure, which is formed because of the negative charge on the glass and bubble surface, causes the formation of a stable film, thereby restricting the particle–bubble attachment (Mukherjee, 2004).

In addition to the experimental attempts to measure the particle–-bubble encounter, several researchers took the advantage of numerical methods including computational fluid dynamics (CFD) to calculate the E_c (Koh et al., 2000; Koh and Schwarz, 2003; Nguyen et al., 2006; Wierink et al., 2009; Liu and Schwarz, 2009a,b; Firouzi et al., 2011). In spite of the usefulness of CFD approach in estimating the particle–-bubble encounter, there is a notable lack of investigation in this regard because it requires deep knowledge of fluid dynamics. The numerical methods, however, have severe constraints in turbulent flow conditions. Moreover, multiphase continuum methods, available in CFD codes, fail to accurately capture the bulk particulate behavior as well as the interaction between the fluid and the particulates. For this reason, discrete element method (DEM) is used as a useful means of simulating particulate systems for some complex industrial applications. However, the Newtonian particle methods used in the conventional DEM restricts their usefulness to particle–only systems. It is one of the main reasons that researchers combine the fluid modeling strengths of CFD with the particle prowess of DEM, sometimes called the combined continuum and discrete model (CCDM) or CFD + DEM. Therefore, the restrictions involved in the measurement and calculation of particle–bubble encounter enforced a large number of researchers to utilize the simplified analytical methods.

Currently, more than ten analytical models are available for prediction of the E_c (Sutherland, 1948; Gaudin, 1957; Flint and Howarth, 1971; Reay and Ratcliff, 1973; Anfruns and Kitchener, 1977; Weber and Paddock, 1983; Dukhin, 1982, 1983; Dobby and Finch, 1987; Yoon and Luttrell, 1989; Schulze, 1989; Dai, 1998; Heindel and Bloom, 1999; Nguyen and Schulze, 2004). These analytical models contain many parameters to estimate the particle–bubble encounter efficiency. The effective parameters can be listed as the fluid flow regime (Stokes, intermediate and potential flows), the degree of mobility of the bubble surface, particle size and its density, dispersion energy and turbulence of the cell, the effective forces (interceptional, gravitational and inertial forces), bubble size and its velocity as well as particle trajectory. Estimation of E_c using analytical models with and without considering the cell turbulence has been adequately elaborated in many studies. Therefore, a brief critical introduction of these models focusing on their drawbacks is presented in this paper and more details can be found in other review articles (Schulze, 1989; Dai et al., 2000; Ralston et al., 2002). The generalized Sutherland equation (GSE) proposed by Dukhin has been accepted as one of the accurate analytical models with respect to taking the particle inertial effect into account (Dai, 1998; Pyke et al., 2003; Duan et al., 2003; Miettinen, 2007). It was also utilized in several studies as one of the accurate analytical models with respect to determining the particle–bubble encounter interactions (Newell and Grano, 2006; Basarova et al., 2010a,b; Jiang et al., 2010; Kouachi et al., 2010; Karimi et al., 2014; Popli, 2017). However, recent numerical results demonstrate that the GSE theory does not correctly predict the particle–bubble encounter probability for every particle size because of its poor estimation of collision angle due to disregarding the microhydrodynamics effects (Nguyen and Nguyen, 2009; Liu and Schwarz, 2009a,b; Firouzi et al., 2011).

The term of microhydrodynamics was initially defined by Batchelor (1976). It deals with predicting macroscopic properties of suspensions from the microscopic behavior of individual and interacting particles in the suspending fluid. Particle motions, as well as suspension microstructure, are the key points of the microhydrodynamic in terms of studying bounded and unbounded flows in the Stokes flow regime (Davis, 1993). Microhydrodynamics effect which is also referred to as short–range hydrodynamic interactions (SRHI) by Happel and Brenner (1963) arises from the water flow in the thin liquid film separating the particle–bubble. The microhydrodynamic interaction at distances comparable to the particle radius causes the particle trajectory to deviate from the liquid flow line. Liu and Schwarz (2009a,b) highlight the significance of microhydrodynamics in particle–bubble encounter efficiency by incorporating a dynamic resistance function. They compared their predicted E_c for quartz and galena particles (d_p = 12–60 μm), for Stokes number of 0.01 to 0.76, mobile bubble and Reynolds number of 151 with the experimental data published by Dai (1998), as well as the predictions of the GSE model, and the numerical methods with and without microhydrodynamics effect. It was reported that the microhydrodynamics effect is significant for quartz and galena particles coarser than 55 and 30 μm, respectively. The impact of microhydrodynamic on particle–bubble encounter was examined by Dukhin (1983), who introduced the drag coefficient with respect to the Stokes drag on a solid sphere moving perpendicularly toward a mobile bubble surface; this was later modified by Dai (1998). Nguyen et al. (2006) improved the theory by considering vertical and parallel directions of particle motion into the modeling of the particle–bubble encounter. Grammatika and Zimmerman (2001) explained the microhydrodynamic mobility problem in flotation as the linear relation between the moments of the surface traction on a rigid particle. The effect of microhydrodynamics on particle–bubble encounter is discussed in more detail in the following sections.

The analytical models whether under quiescent (laminar) or turbulent conditions inadequately estimate the particle–bubble encounter due to many simplified assumptions such as spherical shape of particles and bubbles, homogeneous and isotropic characteristics of turbulence and fluid parameters.

Particle–bubble encounter efficiency is one of the parameters used to mathematically describe flotation kinetics. In the case of mechanically–agitated flotation cells (Duan et al., 2003) or Jameson cells (You et al., 2017), the flotation rate constant depends on the cell volume, the degree of turbulence, air flowrate, particle–bubble encounter, attachment and detachment efficiencies. A similar mathematical model was also developed for a flotation column (Ralston, 1992; Jameson et al., 1977) in which mixing of slurry is much less intense than that in the case of mechanically–agitated flotation cells. It is very important to highlight that this approach has been experimentally validated in laboratory flotation cells but not industrial flotation cells. Therefore, the limitations of this approach in designing industrial flotation cells are unknown and need to be addressed in future research. Thus, the future research has the potential to provide insights into optimal volume of industrial flotation cells to maximize the flotation performance. The future work needs to be focused on using this approach to study not only intermediate particle flotation but also fine and coarse particle flotation in various flotation cells. Required future research based on the identified knowledge gap in this review paper has been recommended in Section 5 of this paper.

Despite excellent reviews on modeling of particle–bubble encounter