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Isotropic turbulence surpasses gravity in affecting bubble-particle collision interaction in flotation

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ABSTRACT

Turbulence and mechanical flotation cells have been the workhorse of the mining industry to process the high tonnage but low-grade ores for more than a century. However, our quantitative understanding of the effect of turbulence on flotation is still limited. Here we theoretically investigate the bubble-particle collision in flotation in homogeneous isotropic turbulence using the correlation method. We show a novel paradigm that isotropic turbulence can surpass gravity in affecting bubble-particle collision in flotation. Specifically, motions of particles of micrometer sizes and bubbles of millimeter sizes are described using the Basset-Boussinesq-Oseen equation. The drag forces on particles and bubbles are calculated using Stokes' law with a particle-size correction factor and Allen's law, respectively. The correlation method is applied to determine bubble and particle velocity variances and covariances. The collision kernel is then calculated, taking into account the effects of turbulence acceleration and shear, and gravity of the bubble-particle system. We compare our collision model with the available models and investigate the influence of bubble and particle sizes, particle density and dissipation rate of turbulent kinetic energy on the collision kernel. The results show that the bubble-particle collision kernel increases with increasing bubble and particle sizes, and dissipation rate of turbulent kinetic energy. Importantly, turbulence can significantly enhance the collision efficiency, exceeding the ideal rate of collision by gravity and leading to the turbulence collision efficiency greater than unity.

1. Introduction

Different types of flotation cells have been invented since the invention of froth flotation (Lynch et al., 2010; Nguyen and Schulze, 2004). Of these cells, the mechanical cells have dominated the industry since the beginning. It is very unlikely that these mechanical cells are going to be replaced by different cell types because of the demand to process the high tonnage of low-grade ores which has led to the current use of very large cells ($> 500\ m^3$). Special cell designs such as flotation columns, Microcels and Jameson cells fulfill the special need of the industry for coal flotation or special cleaning circuits.

In a mechanical flotation cell, air is introduced into the cell near the impeller to form fine bubbles of millimeter size under the mixing effect of the impeller. The rising fine bubbles collect and carry hydrophobic particles (valuable particles) to form a froth layer and exit to the launders while hydrophilic particles (gangue particles) sink to the bottom of the cell to be rejected (Napier-Munn and Wills, 2006). A turbulent flow is claimed to be beneficial to bubble-particle collision and attachment. However, the turbulent effect can cause coarse particles with a high inertia to detach from bubbles, which decreases the flotation efficiency. Three major bubble-particle interaction subprocesses, namely, collision, attachment, and detachment, can be treated separately since their governing forces are independent of each other (Nguyen and Schulze, 2004).

In the literature, the model development of bubble-particle collision in turbulent flow is very limited when compared to that of droplet-droplet collision (Hu and Mei, 1997; Panchev and Haar, 1971; Pinsky et al., 2006; Saffman and Turner, 1956; Wang et al., 1998) or/and particle-particle collision (Abrahamson, 1975; Alipchenkov and Zaichik, 2003; Ayala et al., 2008; Kruis and Kusters, 1997; Meyer and Deglon, 2011; William and Crane, 1983; Yuu, 1984) in gas-particle flows. We note that these available models developed by considering the similar sizes of droplets or solid particles are physically inconsistent with bubble-particle interactions in flotation which have different sizes with different orders of magnitudes (Meyer and Deglon, 2011). Saffman and Turner (1956) derived a collision model of droplets in the turbulent cloud for the limit of small particles which are perfectly correlated with the surrounding carrier fluid. Abrahamson (1975) suggested a model

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Nomenclature

Small alphabet letters

a	numerical constant in Eq. (8)
a_b	a factor defined by Eq. (23)
a_p	a factor defined by Eq. (17)
b	numerical constant in Eq. (8)
b_b	a factor defined by Eq. (24)
b_p	a factor defined by Eq. (18)
d_b	bubble diameter
d_p	particle diameter
f_b	bubble size correction factor
f_p	particle size correction factor
g	gravitational acceleration
k	numerical constant in Eq. (8)
n_b	bubble number concentration
n_p	particle number concentration
t	time
u_0^2	mean square intensity of turbulence

Capitalized alphabet letters

Ar	particle Archimedes number
Ar_*	bubble Archimedes number
E_c	collision efficiency
$\mathbf{F}_{Db}$	drag force acting on a bubble
$\mathbf{F}_{Dp}$	drag force acting on a particle
$\mathbf{J}$	particle flux vector
L_L	Lagrangian integral length scale
M	frequency
Mo	Morton number
N_c	rate of bubble-particle collision
N_{ci}	ideal rate of bubble-particle collision
R	the sum of bubble and particle radii
R_b	bubble radius
R_p	particle radius
Re_b	bubble Reynolds number
Re_p	particle Reynolds number
T_{fL}	Lagrangian integral time scale
$\mathbf{V}$	fluid velocity
$\mathbf{V}_b$	bubble velocity
$\mathbf{V}_p$	particle velocity
V_{bz}	terminal velocity of bubble rising
V_{pz}	terminal velocity of particle settling
V_K	Kolmogorov velocity scale
$\mathbf{W}$	bubble-particle relative velocity vector

Greek letters

Γ	bubble-particle collision kernel
ε	turbulence dissipation rate
λ_K	Kolmogorov length scale
μ	dynamic viscosity of a fluid
ν	kinematic viscosity of a fluid
ρ	fluid density
ρ_b	bubble density
ρ_p	particle density
σ	surface tension of gas-liquid interface
τ_K	Kolmogorov time scale
ω	angular frequency

for the limit of very high inertial particles whose velocities are completely uncorrelated with the surrounding carrier fluid. Since the particle density is assumed to be much larger than that of the carrier fluid, these models are not applicable to the particle-liquid system where the liquid density has the same order as the particle density. Yuu (1984) developed an expression for the fluctuating relative velocity of two inertial particles taking into account the relative velocity between fluid and particle, and the added mass effect experienced by solid particles in a liquid system. He found that the collision due to the spatial variation of turbulence was the predominant factor for small inertial particles in a water stream. However, the particle motion relative to water still increases the collision rate by about 20%.

There have been many deterministic models for bubble-particle collision efficiency available in the literature (Dai et al., 2000; Nguyen et al., 2016). In order to investigate the influence of microturbulence on bubble-particle collision in flotation, a stochastic approach to modeling turbulent flows is highly necessary (Nguyen et al., 2016). Schubert et al. (Schubert, 1996; Schubert, 1999; Schubert and Bischofberger, 1978, 1998; Yoon, 2000) were the first to consider and quantify the effect of turbulence in flotation. Typically, turbulence was considered to affect the collision rate between bubbles and particles in flotation. Modifying the collision rate derived for the particle-particle interaction by Abrahamson (Abrahamson, 1975), Schubert (Schubert, 1999) obtained the following expression for the bubble-particle collision frequency (i.e., the number of collision per unit volume and time):

$$\Gamma = 5n_p n_b (R_p + R_b)^2 \sqrt{\overline{v_p'^2} + \overline{v_b'^2}} \tag{1}$$

where n_p and n_b are the particle and bubble number concentrations, respectively, R_p and R_b are the particle and bubble radii, $\overline{v_p'^2}$ and $\overline{v_b'^2}$ are the root-mean-square values of the turbulent velocity fluctuations of the particles and bubbles, respectively, relative to the turbulent fluid velocity. Using Kolmogorov's theory of isotropic turbulence, these velocities can be connected with the rate of dissipation energy, ε, within the flotation cell, giving the following equation (Schubert, 1999):

$$\overline{v_i'^2} = 0.33 \frac{\varepsilon^{4/9} (2R_i)^{7/9}}{(\rho/\mu)^{1/3}} \left(\frac{\rho_i - \rho}{\rho} \right)^{2/3} \tag{2}$$

where ρ is the liquid density, μ is the liquid viscosity and the index "i" can be "p" or "b" for particles and bubbles, respectively. Many other expressions for the turbulent collision rates between particles or droplets are recently reviewed (Meyer and Deglon, 2011). Schubert et al.'s approach to quantifying the effect of turbulence in flotation was reapplied and re-analyzed by a number of researchers (Jameson et al., 2007; Pyke et al., 2003; Yoon, 2000). In these studies, the effect of turbulence on flotation was incorporated into the bubble-particle collision frequency which is an important term of the rate constant of flotation kinetics in a mechanical cell. Specifically, turbulence has not been considered in predicting the bubble-particle collision efficiency. The available models for the collision efficiency are based on the deterministic collision interaction unaffected by the stochastic interactions with turbulence.

In the present study, we focus on modeling the bubble-particle collision rate and efficiency by taking into account the dependence of colliding particle and bubble velocities through the covariance of bubble-particle fluctuating velocities. We apply the correlation method to derive new expressions for the bubble-particle velocity covariance and related models.