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A general quantification method for addressing stereological bias in mineral liberation assessment in terms of volume fraction and size of mineral phase

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A B S T R A C T

Mineral liberation assessment of ore particles generally involves two-dimensional (2D) measurement of particle cross-sections, and this 2D measurement inevitably results in error due to stereological bias. This bias is considered to be strongly influenced by the internal mineral structure of the particles, but its overall characterization, though investigated conceptually, has hitherto lacked a quantitative foundation. In this study, a systematic numerical study was conducted on particles with an internal mineral structure based on Voronoi modeling, in which three-dimensional degrees of liberation were compared with their 2D counterparts, for random particle cross-sections. In this way, for the first time, overall quantitative characterization of stereological bias, in terms of the volume fraction and size of the mineral phase, was achieved.

1. Introduction

In measuring the degree of mineral liberation of ground ore, two-dimensional (2D) particle cross-sectional measurement inevitably suffers from *stereological bias*, which results in overestimation of the degree of liberation. As illustrated in Fig. 1(a), a composite particle can be misidentified as a liberated particle in a 2D cross-section, leading to stereological bias. Stereological bias has attracted attention since Gaudin pointed out its importance in 1939 (Gaudin, 1939). X-ray computed tomography (CT) (Lin and Miller, 1996; Miller et al., 2009; Videla et al., 2007; Miller et al., 2003; Gay and Morrison, 2006) is effective for obtaining bias-free three-dimensional (3D) liberation information when assessing minerals with large differences in specific gravity. However it has not been commonly used in mineral processing practice, due to limitations in resolution, assessment of minerals with similar densities, and speed (i.e., number of practically measurable particles). A recently developed method, combining X-ray CT and scanning electron microscopy/energy-dispersive X-ray spectroscopy (SEM/EDX)-based analysis, overcomes (or at least reduces) the similar-density limitation in mineral assessment (Reyes et al., 2017), but further technical refinement is necessary to overcome the other limitations for practical use. Serial sectioning (Lätti and Adair, 2001; Schneider et al., 1991; Miller and Lin, 1988), commonly used in cytology, physiology, and so on, offers unbiased information, but is not suitable for practical use in mineral processing, due to limitations in speed and cost (i.e., number of practically measurable particles). Therefore, thus far, for practical purposes, detailed information of mineral phase geometry is obtained by cross-sectional analysis using automated SEM/EDX-based analysis (e.g., using a mineral liberation analyzer (MLA) (Fandrich et al., 2007; Sandmann, 2015), quantitative evaluation of minerals by scanning electron microscopy (Gottlieb et al., 2000), or a TESCAN integrated mineral analyzer (TESCAN, 2012). Therefore, developing *stereological correction*, which predicts the relevant bias and transforms the measurable 2D degree of liberation into its three-dimensional (3D) counterpart, is an important research aim.

Various stereological correction methods have been developed since Gaudin proposed a simple method to estimate the 3D degree of liberation by dividing the 2D degree of liberation by an empirical coefficient (Gaudin, 1939). Miller, King, and their colleagues have transformed 2D liberation distributions into their 3D counterparts using an empirically obtained kernel function (Miller and Lin, 1988; King and Schneider, 1998). Barbery mathematically calculated the 3D liberation distribution through a combination of texture models (e.g., Poisson mosaic) of mineral phases and a particle shape model based on the linear liberation assessment (Leigh et al., 1996). Gay applied fundamental texture analysis of particle cross-sections to stereological correction, by assessing the average distance between two pixels and the average area of a triangle of three pixels, in different phase combinations (Leigh et al., 1997). However, these methods are barely used in practice. As stereological bias is strongly influenced by the internal mineral structure of particles (Lätti and Adair, 2001; Spencer and Sutherland, 2000), the versatility of such methods (i.e., their

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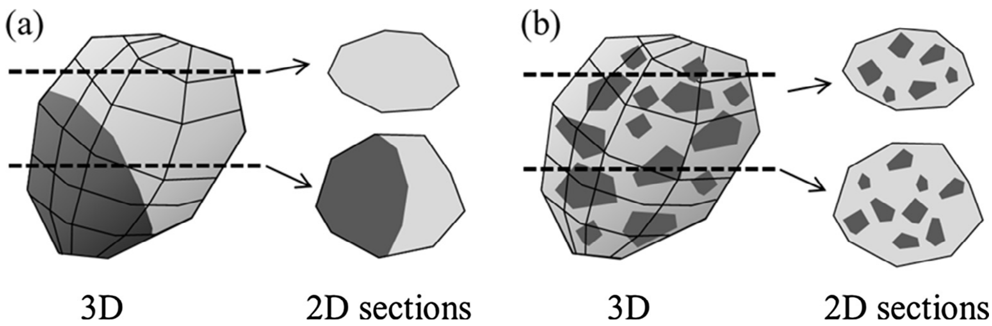

Fig. 1. Difference between the actual 3D state and 2D cross-sectional observation in the case of particles with (a) relatively simple internal structure, and (b) relatively complex internal structure.

applicability to various types of particle systems) is of essential importance, but such versatility has yet to be systematically studied, let alone achieved. The main reason for this, in the opinion of the authors, is that an overall characterization of stereological bias has not been achieved in the first place. A method based on a new conception has been developed, in which stereological bias and the relevant 2D texture features are correlated through an thorough simulation of various types of binary particles, and the stereological bias is predicted by assessing the 2D texture features of the targeted ore samples (Ueda et al., 2016a,b, 2017). This method may be made versatile, in the manner discussed above, through an understanding of the overall character of stereological bias.

Fig. 1 illustrates the effect of the particle internal structure on stereological bias. The bias is large for a particle containing large mineral grains Fig. 1(a)), and small for a particle composed of small mineral grains dispersed throughout the particle matrix Fig. 1(b)). The overall character of the relationship between the particle internal structure and the bias was conceptually illustrated by (Spencer and Sutherland, 2000) Fig. 2. This illustration is informative, and may be the most accepted conceptualization of the relationship. However, its horizontal axis is merely conceptual, and it is unclear which parameter should be taken into account. Moreover, to the authors' knowledge, quantitative and systematic follow-up studies are lacking. Then, in quantifying the overall character of stereological bias, this study investigates two parameters: the volume fraction and size of the mineral phase. Understanding this overall character may lead to a new correction method, elucidate and improve the applicability limits of the existing methods, and provide a criterion for determining whether or not a given correction method is required on site. Such an understanding will also enhance the practical value of results from previous studies on this topic.

A numerical analytical approach is effective for studying stereological bias, because it can freely design the particle shape and internal structure, enabling systematic study; and can calculate accurate

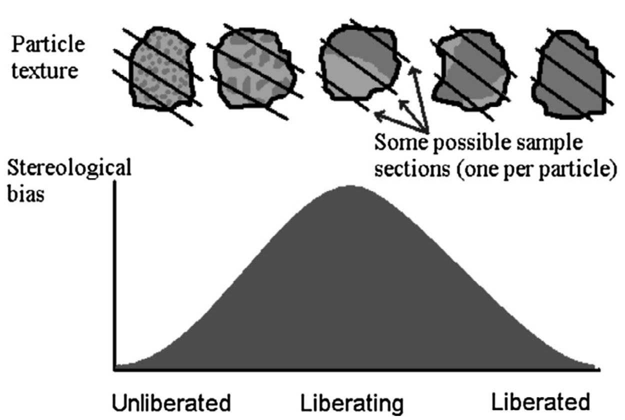

Fig. 2. Conceptual diagram of the overall character of the relationship between the particle internal structure and stereological bias proposed by Spencer and Sutherland (2000).

liberation information in 2D and 3D, which is both costly and time-consuming to calculate experimentally. Generally, in numerical analysis, a 3D particle with a certain shape and an internal structure is modeled, its 3D degree of liberation is calculated, a cross-section is obtained, and the 2D degree of liberation is calculated. The stereological bias is then computed by comparing the degrees of liberation in 3D and 2D. To the authors' knowledge, the pioneering research in this field is the single particle analysis using PARGAMON (Miller and Lin, 1987), where a particle is designed through a process resembling nucleation and growth, with shape factors and randomly selected transformation angles around the coordinate axes; and the mineral phases are modeled by similar processing of the particle, but with designated numbers and volume content. Gay (1999, 2004) took another approach, in which the 3D mineral phase structure (e.g., Boolean) is modeled, the structure is masked by an randomly shaped particle mold, and particles with a random internal structure are created. Gay's method is effective in designing particles with various compositions (0–100%) of the mineral of interest based on a certain texture rule, and the result is interpreted as though the particles are created by randomly breaking a larger rock, where all the mineral phases have the same and isotropic mechanical characteristics. Hilden and Powell (2017) developed a numerical simulation method for the liberation of multi-phased particle systems, based on Gay's modeling. Vassiliev et al. (2008) investigated the effect of crushing types on the liberation state by modeling three types of crushing (random crushing, cracking within the gangue phase, and preferential breakage along phase boundaries), using the Poisson polyhedra mosaic on the Voronoi diagram mineral structure. In the present study, Gay's method is employed to model binary particles with a random composition, and the Voronoi method is employed to model the mineral structure. Detailed information on the modeling of the mineral structure and particle shape is presented in Refs. (Leigh et al., 1996; Serra, 1982; Barbery, 1992; Barbery and Leroux, 1988; Ueda et al., 2017b).

In this study, in order to understand the overall character of stereological bias in terms of the volume fraction and size of the mineral phase, a series of numerical simulations were conducted, in which various types of single spherical binary particles were created using Gay's method, and the 3D liberation information for each particle was compared with the 2D information of a randomly selected cross-section.

2. Methodology

2.1. Binary particle modeling

First, we present a brief overview of the binary particle modeling procedure, before explaining specific models in the subsequent sections.

A spherical particle Fig. 3(a) and 3D mineral structure based on the Voronoi method Fig. 3(b) are prepared.

By carving out a sphere from the mineral structure, a binary spherical particle is obtained Fig. 3(c).

A cross-section at a random position is obtained, and its 2D texture is observed Fig. 3(d).

Note that a spherical particle is used in this study because