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A numerical method to simulate turbulent cavitating flows

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ABSTRACT

The objective of this paper is to develop a numerical method for simulating multiphase cavitating flows on unstructured grids. The multiphase medium is represented using a homogeneous mixture model that assumes thermal equilibrium between the liquid and vapor phases. We develop a predictor–corrector approach to solve the governing Navier–Stokes equations for the liquid/vapor mixture, together with the transport equation for the vapor mass fraction. While a non-dissipative and symmetric scheme is used in the predictor step, a novel characteristic-based filtering scheme with a second order TVD filter is developed for the corrector step to handle shocks and material discontinuities in non-ideal gases and mixtures. Additionally, a sensor based on vapor volume fraction is proposed to localize dissipation to the vicinity of discontinuities. The scheme is first validated for simple one dimensional canonical problems to verify its accuracy in predicting jump conditions across material discontinuities and shocks. It is then applied to two turbulent cavitating flow problems – over a hydrofoil using RANS and over a wedge using LES. Our results show that the simulations are in good agreement with experimental data for the above tested cases, and that the scheme can be successfully applied to both RANS and LES methodologies.

Introduction

Cavitation refers to the formation of vapor when pressure in a liquid drops below vapor pressure. The importance of understanding cavitation lies in its occurrence in a wide variety of applications such as valves, injectors and propulsor blades. The numerical simulation of cavitating flows is inherently challenging since these flows possess a wide range of length and time scales. Additionally, the formation of vapor is often followed by growth of vapor cavities which not only vary in size but also form and collapse at different rates, making their prediction difficult.

The most commonly used physical model to simulate cavitating flows is the homogeneous mixture model. It treats the mixture of water and vapor as a single compressible fluid, and solves a separate transport equation for the mass fraction of vapor (Shin et al., 2003; Kunz et al., 2000; Ahuja et al., 2001; Schnerr et al., 2008; Liu et al., 2004; Saito et al., 2007; Seo and Lele, 2009; Seo et al., 2008; Singhal et al., 2002; Senocak and Shyy, 2002; Adams and Schmidt, 2013; Schmidt et al., 2009). The key differences between commonly used physical models lie in the constitutive equation of state and the mass transfer model. Frikha et al. (2008) provide a review of the different mass transfer models used. Almost all of the simulations mentioned above have used the RANS methodology. However in recent times, DES and LES are also being considered as viable options (Arndt et al., 2000; Bensow and Bark, 2010; Dittakavi et al., 2010; Kinzel et al., 2007; Ji et al., 2013; Wang and Ostoja-Starzewski, 2007). Also, most past simulations invoke the isothermal assumption for cavitation in water. It is known that this assumption is not valid for thermosensitive fluids like cryogenic fluids where an energy equation needs to be solved (Hosangadi and Ahuja, 2005; Zhang et al., 2008; Goncalvès and Patella, 2010). In this study, we use the homogeneous mixture approach with a non-barotropic equation of state for water. In order to maintain a general framework, we have solved an energy equation. In the current investigation, we have focused on hydrodynamic cavitation. The method however, can be applied to thermosensitive fluids as well. The latent heat of evaporation is not considered in this study. Although the specific latent heat of evaporation varies from low values near the critical point to appreciable values near 1 atm and 25 °C, the mass of vapor produced by cavitation is small with respect to the mass of liquid; the amount of latent heat absorbed by vapor formation is therefore negligible. In the examples considered, the vapor mass fraction does not exceed 0.0003.

A turbulent cavitating flow has a broadband spectrum which requires non dissipative numerical schemes (Mahesh et al., 2004; Hou et al., 2005) to represent small scales accurately. However, non-dissipative schemes can become unstable at high Reynolds

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numbers. Furthermore, cavitation is characterized by large gradients in density and strong pressure waves formed during vapor cloud collapse. Accurate representation of turbulence in the presence of these strong gradients is a significant challenge and requires appropriate discontinuity capturing methods. Classical monotonic discontinuity-capturing methods are too dissipative and not suitable for turbulent simulations. Modern discontinuity capturing methods like total variation diminishing (TVD) schemes, essentially non-oscillatory (ENO) schemes and monotone upstream-centered schemes for conservation laws (MUSCL) typically incur higher computational cost for achieving higher order of accuracy in the vicinity of discontinuities. Further, these schemes require special treatment near boundaries (Yee et al., 1999). Yee et al. (1999) proposed a class of filters called 'characteristic filters', that add the dissipative part of a traditional shock capturing method to a non-dissipative base scheme. They developed this method for ideal gases on structured grids; Park and Mahesh (2007) proposed an extension to unstructured grids. Numerical boundary conditions for these filters can be same as the existing base schemes, which is an added advantage. Further, the characteristic filter can be applied to the solution once, after a full time step, and hence is considerably cheaper than the TVD, ENO and MUSCL schemes (Lo et al., 2010). A simple linear filter was first proposed by Gustafsson and Olsson (1995), which provides a linear second order dissipation. Yee et al. (1999) then used a second order non-linear TVD filter that takes into account the different wave characteristics of the Euler equations. Both lower order TVD or higher order ENO/WENO type terms can be used as characteristic filters. Lo et al. (2010) observed that WENO type filters perform marginally better than lower order TVD filters and also found WENO type filters to be insensitive to the tunable parameters that appear in the shock capturing scheme. Both Park and Mahesh (2007) and Lo et al. (2010) observed that the original combination of TVD filter and Harten's artificial compression method (ACM) switch (Harten, 1983) proposed by Yee et al. (1999) was not able to distinguish between turbulent fluctuations and shocks, and hence proposed modified switch terms.

In this paper, we extend the characteristic based filtering method to non ideal gases and a mixture of fluids to simulate multiphase cavitating flows on unstructured grids. A predictor–corrector method is used where the predictor step is non-dissipative and the corrector step computes the jump conditions across the discontinuities. The dissipation is spatially localized to reduce dissipation away from the discontinuities. We propose an additional modification to this localization term applicable in multiphase flows. The governing equations are spatially Favre filtered for LES. The additional terms arising out of spatial filtering are modeled using a Dynamic Smagorinsky model. The paper is organized as follows. Section 'Governing equations' outlines the governing equations along with the source terms for evaporation of water and condensation of vapor. Section 'Numerical method' discusses the predictor–corrector algorithm along with the spatial and temporal discretization schemes. The characteristic based filtering applied as a corrector step is also discussed in this section. Validation simulations are presented in Section 'Results', and a brief summary in Section 'Summary' concludes the paper.

Governing equations

We use a homogeneous mixture model that assumes thermal and mechanical equilibrium between the phases i.e. there is no slip velocity or temperature difference between the phases. Also, surface tension effects are ignored. The constituent phases are treated as a single compressible fluid whose density

$$\rho = \rho_l(1-\alpha) + \rho_g\alpha, \tag{1}$$

where ρ_l is the density of liquid and ρ_g is the density of vapor. α is the vapor volume fraction which is related to the vapor mass fraction (Y) by

$$\rho_l(1-\alpha) = \rho(1-Y) \quad \text{and} \quad \rho_g\alpha = \rho Y. \tag{2}$$

The governing equations are the Navier–Stokes equations along with a transport equation for the mass fraction of vapor:

$$\begin{aligned}
\frac{\partial \rho}{\partial t} &= -\frac{\partial}{\partial x_k}(\rho u_k),\\
\frac{\partial \rho u_i}{\partial t} &= -\frac{\partial}{\partial x_k}(\rho u_i u_k + p\delta_{ik} - \sigma_{ik}),\\
\frac{\partial \rho Y}{\partial t} &= -\frac{\partial}{\partial x_k}(\rho Y u_k) + S_e - S_c,
\end{aligned} \tag{3}$$

where ρ, u_i and p are density, velocity and pressure respectively of the mixture. For energy transport, both total energy and internal energy forms are considered. Their relative merits and demerits are discussed in Section 'Multiphase non cavitating shock tube'. The internal energy form is used for the results shown unless specified otherwise.

$$\begin{aligned}
\frac{\partial E_T}{\partial t} &= -\frac{\partial}{\partial x_k}\{(E_T + p)u_k - \sigma_{ik}u_i - Q_k\},\\
\frac{\partial \rho e_s}{\partial t} &= -\frac{\partial}{\partial x_k}(\rho e_s u_k - Q_k) - p\frac{\partial u_k}{\partial x_k} + \sigma_{ik}\frac{\partial u_i}{\partial x_k}.
\end{aligned} \tag{4}$$

Here E_T and e_s are total energy and internal energy respectively.

$$\begin{aligned}
&\rho e_s = \rho_l e_l(1-\alpha) + \rho_g e_g \alpha, \text{ where}\\
&e_l = C_{vl}T + \frac{P_c}{\rho_l},\\
&e_g = C_{vg}T,\\
&\rho e_s = \rho C_{vm}T + \rho(1-Y)\frac{P_c K_l}{p+P_c} \text{ and}\\
&E_T = \rho e_s + \frac{1}{2}\rho u_k u_k.
\end{aligned} \tag{5}$$

Here, e_l and e_g are the internal energies of liquid and gas respectively. C_{vl} and C_{vg} are the specific heats at constant volume for liquid and vapor respectively and C_{pl} and C_{pg} are the specific heats at constant pressure. The system is closed using a mixture equation of state based on stiffened equation of state for water and ideal gas equation for vapor.

$$p = Y\rho R_g T + (1-Y)\rho K_l T\frac{p}{p+P_c}. \tag{6}$$

Here, R_g = 461.6 J/Kg K, K_l = 2684.075 J/Kg K and $P_c = 786.333 \times 10^6$ are constants associated with the equation of state of vapor and liquid. The density and speed of sound predicted by the stiffened equation of state is compared with the National Institute of Standards and Technology (NIST) data in Fig. 1(a) and a good agreement is observed. However the stiffened equation of state underpredicts the value of specific heat at constant volume C_{vl} (predicts it to be 1500.3 J/Kg K as opposed to the NIST value of 4157.4 J/Kg K). This is not seen as a serious drawback in the current study because, heat transfer effects within the liquid phase are small in hydrodynamic cavitation at ambient pressure and temperature. The proposed numerical method however can be applied to more complicated equations of state for water like the Tait equation of state. The stiffened equation of state is chosen due to its simplicity. Since internal energy is a function of both pressure and temperature, we need to obtain these variables using Eqs. (5) and (6). Solving these two equations simultaneously yields a quadratic equation $ap^2 + bp + c = 0$, where