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A capillary flow model for filtration

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ABSTRACT

A filtration model has been developed by considering the two- and three-phase flows through a filter cake during the cake formation and drying cycle times, respectively. Assuming that the cake consists of well-defined capillaries of identical radius, it is possible to predict filtration kinetics from first principles using the Navier-Stokes equation and capillary pressures. The model parameters have been determined by fitting experimental data to the model using capillary radius and slip length as adjustable parameters. It has been found that use of hydrophobizing reagents greatly improves filtration kinetics and reduces cake moistures, which may be attributed to increases in capillary radius and possibly in slip lengths. Use of a polymeric flocculant also increases filtration kinetics but can cause an increase in cake moisture due to the entrapment of moisture within floc structures.

1. Introduction

In 2016, Canada produced 2.4 million barrels per day (b/d) of oil from the oil sands resources in Alberta, which is expected to grow to 3.67 million b/d by 2030 (Canadian Association of Petroleum Producers, 2013). One of the challenges to meeting the projection is the issues concerning tailings management. At the end of 2013, the industry was using 220 km^2 of the lands to construct tailings ponds. Significant amounts of the tailings are silts and clays that are slow to settle and consolidate, creating serious problems for reclamation. The objective of the present investigation was to develop a filtration model that can lead to a better understanding of the mechanisms involved in dewatering fluid tailings and to finding practicable solutions.

Many investigators developed filtration models using the Darcy's law, which was an empirical model derived in 1856 to describe fluid flow through sand filters. Years later, other investigators derived analytical expressions from the Navier-stokes equation with a few assumptions (Hall, 1956). The Darcy's law is useful to model the two-phase flow through a filter cake during the cake formation time. As shown by Kozeny (1927) and Carman (1997, 1939) the fluid velocity is strongly a function of cake porosity, which in turn varies with particle size, shape, and distribution. The Darcy's law may also be used to model the three-phase flow of water and air during the drying cycle time with due consideration of driving pressure. Wakeman (1976, 1979a,b) introduced a parameter known as *breakthrough pressure* (p_b) to represent the pressure required for air to penetrate a filter cake and induce desaturation (or dewatering). It has been shown that p_b varies with particle size, cake porosity, and surface tension. Carleton and Mackay (1988) added contact angle (θ) as an additional parameter affecting p_b. These investigators assumed that filter cake consists of cylindrical capillaries. Condie et al. (1996) and Condie and Veal (1997) fitted their coal filtration data to the Wakeman model to show that changes in cake porosity caused profound impacts on dewatering but the impact of changing θ was minimal. On the other hand, Brookes and Bethell (1984) showed substantial benefits of using a hydrophobizing dewatering aid for fine coal dewatering. One of the authors of the present communication developed a suite of reagents that can be used to increase θ to improve fine particles dewatering (Yoon, 2005).

The Darcy's law is typically used with the no-slip boundary condition, *i.e.*, the velocity of water is zero at the wall of a capillary. This assumption worked so well for decades. Recent studies showed, however, that water slips on hydrophobic surfaces, with the slip lengths varying with surface hydrophobicity, surface roughness, and shear rate. Measured slip lengths are in the range of tens of nanometers (nm), but the largest values reported in the literature were approximately 1 μm (Lauga et al., 2007). It is well accepted now that the flow around/above

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air bubbles slips, which led to a suggestion that slip on hydrophobic surfaces may be due to the nano-bubbles present on hydrophobic surfaces (Lauga and Stone, 2003). It appears that the slip at solid/liquid interfaces depends on the nature of the water molecules at the interface. Slip should occur if the work of adhesion is less than the work of cohesion of water (Granick et al., 2003). The slip lengths (b) measured in the present work using the method described by Churaev et al. (1984) were found to increase with increasing water contact angles.

The model was derived assuming that a filter cake consists of a bundle of capillaries of identical radius and length, which made it easier to use the Navier-Stokes equation to determine filtrate rate during the cake formation and drying cycle times. The model parameters included slip length, so that the model can be used to study the effects of particle hydrophobicity on filtration rate. A series of dewatering tests was conducted, with the results used to determine the changes in the capillary radius and slip length with time and operating conditions. The results were useful for better understanding the possible changes in cake structure and dewatering mechanisms.

2. Model development

The process of filtration may be subdivided into two cycles, *i.e.*, cake formation and drying. In the former, a filter medium is submersed into an aqueous slurry to form a filter cake, which emerges subsequently to allow air to penetrate the cake and drive the water out of the capillaries to initiate dewatering. Thus, the fluid mechanics involved during the cake formation and drying cycle times may be characterized as two- and three-phase flows, respectively, as described below.

2.1. Two-phase flow

Considering that a filter cake consists of multitudes of cylindrical tubes (capillaries) of identical radius R and length L, the flow of water through a single capillary can be described by the Navier-Stokes equation as follows,

$$0 = -\frac{\partial p}{\partial z} + \mu\frac{1}{r}\frac{d}{dr}\left(r\frac{dv_z}{dr}\right) \tag{1}$$

where p is the pressure of the liquid in the capillary, μ its dynamic viscosity, v_z the fluid velocity in z-direction, and r is the radial distance (Fig. 1a).

Assuming that the pressure gradient ($\partial P/\partial z$) does not change along the z-direction, Eq. (1) becomes,

$$\mu\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial v_z}{\partial r}\right) = \frac{\Delta P}{L} \tag{2}$$

where ΔP is the pressure drop along the capillary of length (L).

Integration of Eq. (2) twice gives,

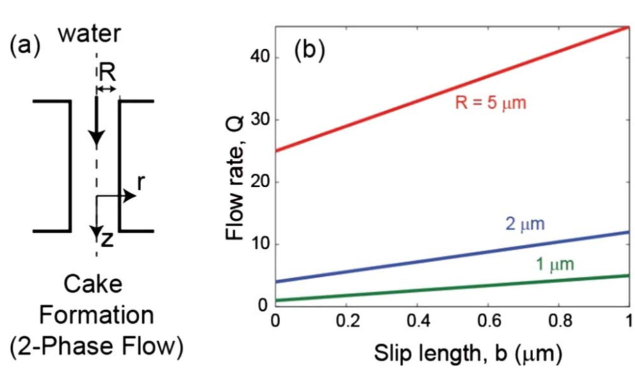

Fig. 1. (a) Two-phase flow of water through a capillary tube; (b) Effect of slip length (b) on flow rate (Q) at different pore radii (R).

$$v_z = \frac{1}{\mu}\frac{\Delta P}{L}\frac{r^2}{4} + C_1 \ln r + C_2 \tag{3}$$

where C_1 and C_2 are two unknowns, which can be determined under specific boundary conditions. Under the no-slip boundary condition, *i.e.*, $v_z = 0$ at $r = R$, Eq. (3) becomes,

$$v_z = \frac{1}{4\mu}\frac{\Delta P}{L}(R^2 - r^2) \tag{4}$$

and

$$v_z = \frac{1}{4\mu}\frac{\Delta P}{L}(R^2 - r^2) + \frac{\Delta P}{\mu L}\left(\frac{Rb}{2}\right) \tag{5}$$

under the slip boundary condition with a slip length b. At $b = 0$, Eq. (5) is reduced to Eq. (4).

Integrating Eq. (5) across the cross-sectional area of a capillary tube, and knowing the fraction (k) of the cross sectional area (A) of a filter cake that is occupied by the capillaries of identical radii of R, one can determine the volumetric flow rate of fluid through a filter cake (Q) as follows,

$$Q = \frac{kA}{\pi R^2}q = \frac{kA}{\pi R^2}\int_{r=0}^{R} v_z 2\pi r dr = \frac{Ak\Delta P}{\mu L}\left(\frac{Rb}{2} + \frac{R^2}{8}\right) \tag{6}$$

Eq. (6) suggests that Q (dV/dt) should increase with b, which should in turn increase with increasing hydrophobicity of the inner walls of the capillaries (Lauga et al., 2007).

Fig. 1b shows plots of Eq. (6) at $R = 1$, 2, and 5 μm. As shown, Q increases with increasing R and b. Although not clearly discernable, the plots show that the benefits of increasing b become greater at smaller radii. At 1 μm, for example, Q doubles at $b = ¼\,R$ and quintuples at $b = R$, which is a powerful message that one can increase filtration rate of fine particles by increasing slip lengths, which can be readily achieved by simply using a hydrophobizing dewatering aid.

The cake thickness L will increase during the cake formation time. One can see that $L = X_s V/A(1-k)\rho_s$, where X_s is the mass of (dry) solids in the cake per volume of filtrate generated and ρ_s is the density of particles. Substituting this for L in Eq. (6) and solving it for ΔP;

$$\Delta P = \frac{8\mu X_s V}{kA^2R^2\rho_s}\frac{1}{1-k}\frac{1}{1+b/4R}\frac{dV}{dt} = \Delta P_c \tag{7}$$

where ΔP_c may be designated as the pressure drop across a filter cake.

The pressure drop across the filter medium (ΔP_m) may be given by the Darcy's equation as follows,

$$\Delta P_m = \mu R_m \frac{1}{A}\frac{dV}{dt} \tag{8}$$

where R_m is the resistance to flow from the medium. The overall pressure drop may then be given as,

$$\Delta P = \Delta P_c + \Delta P_m = \frac{8\mu X_s V}{kA^2R^2\rho_s}\frac{1}{1-k}\frac{1}{1+b/4R}\frac{dV}{dt} + \mu R_m\frac{1}{A}\frac{dV}{dt} \tag{9}$$

Eq. (9) may be rearranged to obtain,

$$\frac{dV}{dt} = \frac{A^2\Delta P}{\mu\left[\frac{8X_s V}{kR^2\rho_s}\frac{1}{1-k}\frac{1}{1+b/4R} + AR_m\right]} \tag{10}$$

which gives the rate of filtration (Q) as functions of the usual operating variables of cake filtration, *i.e.*, pressure drop (ΔP), solids content (X_s), cross sectional area of filter cake (A), cake surface porosity (k), pore (capillary) radius (R), medium resistance (R_m), and slip length (b). Both k and R can be related to particle size, while b can be controlled by dewatering aids. An integral of Eq. (10) gives the volume of filtrate after a given filtration time.

For the liquid in a small pore or the thin liquid film (TLF) confined between two basal surfaces of clay particles, the disjoining pressure (Π) due to surface forces may affect fluid flow. According to the extended