International Journal of Multiphase Flow 59 (2014) 145–159

Contents lists available at ScienceDirect

International Journal of Multiphase Flow

journal homepage: www.elsevier.com/locate/ijmulflow

Different particle-accumulation structures arising from particle–boundary interactions in a liquid bridge

Frank H. Muldoon *, Hendrik C. Kuhlmann

Institute of Fluid Mechanics and Heat Transfer, Vienna University of Technology, Resselgasse 3, A-1040 Vienna, Austria

ARTICLE INFO

Article history:
Received 23 June 2013
Received in revised form 20 October 2013
Accepted 22 October 2013
Available online 9 November 2013

Keywords:
Particle-accumulation structure
PAS
Thermocapillary flow
Marangoni effect
Liquid bridge
Particle–wall interaction
Particle–free-surface interaction
Incompressible flow
Poincare sections

ABSTRACT

The formation of particle-accumulation structures in the flow in a cylindrical liquid bridge driven by the thermocapillary effect is studied. The problem is modeled as an incompressible fluid seeded with a low concentration of small spherical particles which are assumed to have a negligible effect on the flow. The particle motion is modeled by pure advection except for small regions at the free surface and walls where the particle interaction with the free surface and walls is modeled by a hard-wall potential which restricts the motion of the particle within some interaction length from the boundaries. The model yields a wide range of particle-accumulation structures when varying the relative interaction length from 5×10^{-3} to 3×10^{-2}. It is found that in many cases each particle need undergo no more than a handful of returns to the free surface for particle-accumulation structures to arise. It is also found that the shape of the particle-accumulation structures changes qualitatively at certain critical interaction lengths.

1. Introduction

A well accepted model to study the nature of the thermocapillary flow in the floating-zone crystal-growth technique (Hurle, 1994) is what is known in the literature as a *liquid bridge*. In this model the midplane of the full floating zone is replaced by a hot solid wall. The liquid then forms a bridge between the hot and cold solid walls which is held in place by surface tension. As a result of the temperature gradients and the temperature dependence of the surface tension, a thermocapillary (Marangoni) force exists tangent to the free-surface. Under conditions of weightlessness and neglecting any surrounding gas, this force represents the sole force driving the liquid motion. When the temperature difference between the hot and cold walls, assumed to be flat, parallel and concentric, exceeds a critical value the basic axisymmetric toroidal vortex flow becomes unstable to azimuthally propagating three-dimensional waves, called hydrothermal waves (Smith and Davis, 1983; Wanschura et al., 1995), if the Prandtl number is larger than about one. A sketch of this model is shown in Fig. 1.

Schwabe et al. (1996) have shown that under certain conditions seeding particles used for flow visualization can form particle accumulation structures which they called dynamic Particle Accumulation Structures (PAS). It was observed by Schwabe et al. (2007) that the accumulation process is fastest when the density of the particles equals that of the liquid. Since the size of the particles used in the experiments was very small, such density-matched particles should nearly perfectly follow the streamlines and hence starting from an even seeding of particles in the liquid, appreciable changes from this initially constant number of particles per volume of liquid should not occur. Since the deviation of particle trajectories from streamlines due to the difference in density between the particles and the liquid vanishes as the difference in density vanishes, it is evident that density differences are not the primary mechanism causing the particles to deviate from streamlines of the flow in these experiments. Clearly, however, a mechanism causing significant and rapid deviation from streamlines of the flow exists. Hofmann and Kuhlmann (2011) and Kuhlmann and Hofmann (2011) suggested a mechanism by which particles, originally evenly distributed throughout the liquid, can accumulate via a finite-particle-size effect which prevents them from following the streamlines only in a small region near the free surface. Their physical model of PAS formation is based on their particle–free-surface interaction model and the structure of thermocapillary flows, in particular the extreme crowding of streamlines towards the free surface and the presence of closed streamlines and stream tubes near the free surface. Other investigators (Melnikov et al., 2011; Pushkin et al., 2011) have found PAS which they attributed to inertial effects due to a difference between the particle density

* Corresponding author. Tel.: +79313075021.
E-mail address: fmuldoo@me.lsu.edu (F.H. Muldoon).

0301-9322/$ - see front matter © 2013 Elsevier Ltd. All rights reserved.
http://dx.doi.org/10.1016/j.ijmultiphaseflow.2013.10.012

and the density of the fluid. A comparison of the two effects is given in Kuhlmann and Muldoon (2012b). "Phase locking" (Pushkin et al., 2011) has also been advanced as a cause for PAS, but see also Kuhlmann and Muldoon (2012a). It has also been discovered (Muldoon and Kuhlmann, 2013b) that in numerical simulations PAS can arise entirely due to error in a form identical to that observed experimentally. It should be noted that attractors for the particle motion can be created due to the dissipative effect on the particle motion caused by inertia when the particle density differs from that of the liquid (Yarin et al., 1996; Lyubimov et al., 2005). Under the experimental conditions of Tanaka et al. (2006) and Schwabe et al. (2007), however, these effects are too weak, owing to the small particle Stokes number, to explain the rapid time scale on which PAS forms (Kuhlmann and Muldoon, 2012b).

Numerical modeling of this phenomenon has been hindered by the large computational expense of solving the governing equations of the flow, which are commonly taken as the unsteady incompressible Navier–Stokes and energy equations, in addition to equations governing the motion of the particles. Since the volume of the particles compared to that of the fluid is small (generally less than 1%), the problem can be simplified by assuming that the particles have negligible influence on the flow. Nonetheless, the problem remains challenging from a computational viewpoint. This is particularly true for the modeling of the motion of the particle near walls or free surfaces. While some analytical and numerical results have been obtained for highly idealized settings (see e.g. Bart, 1968; Lee et al., 1979; Leal, 1980; Takemura and Magnaudet, 2003; Zeng et al., 2005; Liu and Prosperetti, 2010) there exists no straight-forward particle–boundary interaction model that could readily be implemented without explicitly resolving the length and time scales relevant in such interaction processes.

For that reason we use in the present work an inelastic hard-wall model for the particle motion proposed by Hofmann and Kuhlmann (2011) and Kuhlmann and Hofmann (2011). It contains a single parameter which we refer to here as the interaction length Δ. This parameter should be closely related to the particle radius a. Kuhlmann and Hofmann (2011), Hofmann and Kuhlmann (2011), Kuhlmann and Muldoon (2012b) and Muldoon and Kuhlmann (2013a) used $\Delta = a$. This hard-wall model reduces the parameter space to be considered, but nonetheless, there is a very wide range of particle sizes for which PAS has been observed. In this work, we show through extensive simulations that the form of PAS is extremely sensitive to the interaction length and thus to the particle size at certain distinct values of the interaction length and insensitive over other ranges of interaction lengths. This behavior we attribute to the presence in the flow of regular stream tubes situated at similar length scales of the interaction length from the free surface.

2. Description of the physical problem and the mathematical models

2.1. Description of physical problem

We consider a drop of liquid seeded with small spherical particles with a density matched to that of the liquid. The liquid drop is held in place between two posts by surface tension and surrounded by a passive gas under conditions of zero gravity (Fig. 1). The volume $\mathcal{V}$ of the drop of liquid corresponds to a cylinder with height d and radius R where the aspect ratio is $\Gamma = d/R = 0.66$. The two posts are kept at different temperature $T_0 - \Delta T/2$ and $T_0 + \Delta T/2$, where T_0 is a reference temperature. Due to the temperature gradients along the interface the surface tension varies and gives rise to an interfacial shear stress, i.e., the thermocapillary effect. A gross feature of the induced flow is a basic toroidal vortex where the liquid at the interface moves from the hot towards the cold wall and returns in the bulk. At high enough Reynolds numbers the axisymmetry of the flow is broken and a traveling or a standing hydrothermal wave develops (Leypoldt et al., 2002), if the Prandtl number satisfies Pr $\gtrsim$ 1 (oxide crystals, transparent model liquids). The traveling wave, shown in Fig. 2, rotates with a constant angular phase velocity about the axis of the liquid bridge. As the Reynolds number is further increased, the flow eventually becomes fully turbulent.

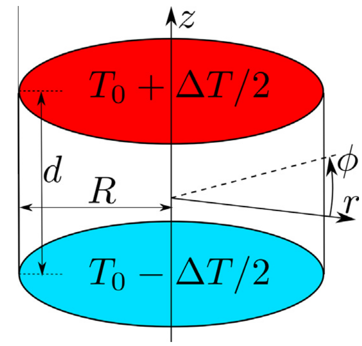

Fig. 1. Geometry of a differentially heated liquid bridge with volume $\mathcal{V} = \pi R^2 d$.

2.2. Mathematical model of flow

The problem is simplified by assuming the surrounding gas is sufficiently dilute such that its effect on the fluid can be neglected and that the liquid is incompressible with constant fluid properties except for the surface tension, which is assumed to depend only on temperature and to have the following form

$$\sigma = \sigma_0 - \gamma(T - T_0). \tag{1}$$

The linear approximation of the surface tension (1) is a good approximation for the typical temperature differences encountered in experimental realizations. As the mean surface tension $\sigma_0 \gg \gamma(T - T_0)$ is usually much larger than the temperature-induced change of the surface tension, the Laplace pressure dominates the flow-induced pressure. Therefore, we consider the limit of vanishing capillary number $\mathrm{Ca} = \gamma\Delta T/\sigma_0 \to 0$ (Sen and Davis, 1982; Kuhlmann, 1989) in which the liquid drop takes a cylindrical shape, regardless of the thermocapillary convection inside the liquid (see also Kuhlmann, 1999).

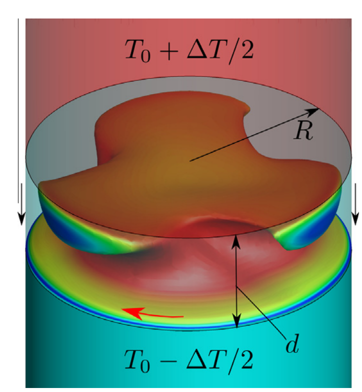

Fig. 2. Three-dimensional view of flow inside liquid a bridge. The three-pronged structure is the rotating hydrothermal wave, visualized by an isosurface of T_0 colored by vertical velocity. $\mathrm{Re} = 1800, \mathrm{Pr} = 4, d/R = 0.66$. Vertical arrows indicate direction of thermocapillary flow along the free surface; red arrow indicates rotation of the hydrothermal wave.