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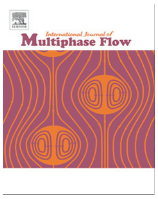

A statistical model for the average volume fraction profile through the mixing zone in turbulent stratified gas–liquid flow

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ABSTRACT

A phenomenological model was developed for the average volume fraction profile through the mixing zone between turbulent gas and liquid in horizontal, layered flow. A probability distribution (PDF) of the height of the interface was defined in terms of a ratio between the interfacial energy (system energy) and the turbulent kinetic energy (driving energy), representing an analogy to the Boltzmann distribution in statistical physics. A potential advantage with this modelling approach is that it offers an alternative to (often inaccurate) entrainment correlations that are used in boundary conditions for dispersion models. A unified PDF and dispersion model was tested against X-ray tomography data, with satisfactory results.

1. Introduction

Mixing or entrainment across gravitationally stable interfaces due to background turbulence is a frequent phenomenon that is important to chemical and nuclear engineering (e.g., with the transport of gas and fluids in pipes), and in the environmental area relating to transport processes (e.g., mixing of atmospheric gases into the ocean). The current work was motivated by gas/liquid or liquid/liquid flow in pipes or channels, where the turbulent kinetic energy is often comparable to the interfacial energy, with a significant entrainment of one fluid into the other in the form of droplets or bubbles (e.g., Rein, 1998). Related scenarios are found in oceanography, with respect to turbulence and waves induced by surface wind (e.g., Fernando, 1991; Wu, 1980), and to entrainment and mixing processes due to turbulence and waves.

Mixing and entrainment may be induced by direct turbulent forcing of the interface (vorticity-interface interaction), or by hydrodynamic instabilities of shear driven interfacial waves (controlled by gravity, capillarity and viscosity) (Fernando, 1991). The details of the mixing process depend on the nature of the forcing (shear driven or turbulence driven), but also the properties of the interface in terms of interfacial tension, rheology, and elasticity which may all be modified if surfactant is present (e.g., Sarpkaya, 1996).

In the context of direct forcing by turbulence, Linden (1973) studied impinging vortex rings on a sharp density interface. At high Richardson number, $Ri \gg 1$, where gravitational stabilization of the interface dominates over the driving kinetic energy, the vortices simply flatten when they approach the interface, so that entrainment by turbulence is less efficient than hydrodynamic instabilities induced by shear. According to Long (1978), the integral-scale eddies cannot play a direct role in the entrainment at high *Ri*, and the eddy-flattening merely causes an energy transfer from the vertical to the horizontal velocity component adjacent to the interface. It appears that the dominant entrainment mechanism in this regime is interfacial wave breaking (Fernando, 1991). In the current study, we focus on the opposite regime where the Richardson number is of order unity, such that entrainment due to turbulence is the dominant mechanism.

DNS analysis of the turbulence–interface coupling have focused mainly on non-entraining cases where the interface stays intact as one connected sheet, and early work focused on the turbulence structure near free surfaces with and without shear (e.g., Lugt and Ohring, 1992; Lombardi et al., 1996; Fulgosi et al., 2003). Guo and Shen (2003) analyzed the statistics and energy budget of the turbulence near a free surface with finite amplitude deformations caused by propagating waves and turbulence-generated surface roughness. A similar non-entraining air/water interface

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was studied experimentally by Smolentsev and Miraghaie (2005) in an inclined open channel, as the flow changed from weak to strong turbulence. For stronger turbulence, the turbulence–interface interaction became dominant over the wave phenomena, and the turbulent structures at the interface had a distinct 3D structure. DNS simulations of disrupting/fragmenting interfaces have now become numerically feasible also in 3D, with focus on jet breakup (e.g., Ménard et al., 2007), and we expect valuable insight to be drawn from such numerical experiments.

There are some important theoretical and experimental works that form the basis of the current phenomenological–statistical approach for the volume fraction profile through the mixing zone. Early experiments on scalar mixing over density interfaces focused on grid generated turbulence (Linden, 1975; Redondo et al., 1996). In these experiments, the density gradient evolved over time with a gradual reduction of the gradient of the average density profile. More recently, Whitehead and Stevenson (2007) and Whitehead (2008) analyzed a similar case in a stirred tank, with focus on the effect of varying Richardson number. A two-layer salt-stratified tank of water was considered, with a Richardson number of about 0.9. The density profile began as a single step and evolved to a smooth mixed profile. It was shown that a universal time dependent profile (in terms of the error function) exists as a function of a similarity variable.

The current probabilistic approach is inspired by Whitehead and Stevensons work, and we show that the volume fraction profile through the mixing zone (analogous to the density profile in their experiments) can be derived from a probability distribution function (PDF). In our case, the flow is statistically stationary with no time development, and we consider immiscible fluids with a sharp interface, rather than a continuous density field.

Brocchini and Peregrine (2001a,b) considered interfacial deformation and breakup under the stabilizing effect of gravity and interfacial tension, and with the destabilizing effect of turbulence. These authors focused on the interfacial stability properties as function of interfacial length scale and turbulent kinetic energy, and discussed the effect of the interfacial Weber and Froude numbers. Wacławczyk and Oberlack (2011) developed ensemble averaged evolution equations for the volume fraction over the interfacial region in turbulent flow, and could relate their volume fraction profile to a Gaussian PDF for the interface position for the time independent, stationary case. The parameters in their PDF were estimated based on the turbulent kinetic energy and dissipation rates.

Based on these earlier works, we resort to a phenomenological–statistical model of the interfacial fluctuations in both non-entraining and entraining regimes, and generalize the model to any viscosity and density ratio and we include viscous damping effects. We formulate a probability distribution function (PDF) for the interfacial location in terms of the ratio between the interfacial energy (including viscous deformation work, gravitational potential energy and interfacial energy due to interfacial tension), and the turbulent kinetic energy near the interface. The variance of this PDF is assumed to scale with this energy ratio so that larger turbulent kinetic energy corresponds to a wider PDF. This choice represents an analogy to the (exponential) Boltzmann distribution in statistical physics, that depends on the ratio between the system energy (energy level of the system – here, the interfacial energy as function of interfacial height) and the driving kinetic energy in the system (kinetic temperature – here, the turbulent kinetic energy). We demonstrate that this analogy generates a model that represents the X-ray data for different flow conditions very well.

An advantage with this energy based approach is that detailed modelling of the different the entrainment mechanisms is not required, also much in the spirit of statistical physics. Clearly, this description can only predict average behavior (here, the volume fraction profile for given average energies), but with the potential advantage of producing a more robust model for application, with only a few model parameters. From the PDF, we obtain the average (and stationary) volume fraction profile through the mixing zone by integration. The continuation of the model into the dispersion zones also fits the data very well, without the need for (often uncertain) entrainment and deposition correlations that we have used earlier (e.g., Skartlien, 2009; Skartlien et al., 2011). Laboratory experiments with X-ray tomography of gas/liquid pipe flow were used to determine the closure parameters in the model.

2. Model development

2.1. Interfacial probability distribution function

The foundation of the current model is a probability distribution function (*PDF*), $P(h)$ for the interfacial height, h relative to the reference level ($h = 0$). The quantity $P(h)dh$ is the probability of finding the interface in the height interval $[h - dh, h + dh]$. We adopt an analogy to the Boltzmann distribution by postulating a phenomenological relation of the form

$$P(h) = C\exp\left(-\frac{\text{System energy}}{\text{Driving energy}}\right), \tag{1}$$

where C is a normalization constant to be determined via closure relations. In our case, the driving energy is the turbulent kinetic energy and a possible contribution from wave energy on length scales comparable to the turbulence length scales. The kinetic energy of the fluid is then taking the role of the (micro-scale) thermal kinetic energy k_BT in the original Boltzmann distribution. The system energy is now the interfacial energy in terms of gravity, interfacial tension and viscous deformation work,

$$\text{Driving energy} = \text{Turbulent kinetic energy} + \text{wave energy}, \tag{2}$$

$$\text{System energy} = \text{Interfacial energy}. \tag{3}$$

The physical content of these terms is developed in the following.

A connected interface simplifies the geometrical relations that are necessary to formulate the interfacial energy, and we expect that the interface is *mostly* connected sufficiently close to the reference level. At larger distances, and for sufficiently large turbulent kinetic energy, we expect that the interface is mostly disconnected in the form of droplets or bubbles, due to the rupture of stretched fluid filaments that are extruded from the interface (Fig. 1). Based on this argument, we define an *interfacial zone* in the interval (h_e^-, h_e^+) around the reference level ($h = 0$) where it suffices to

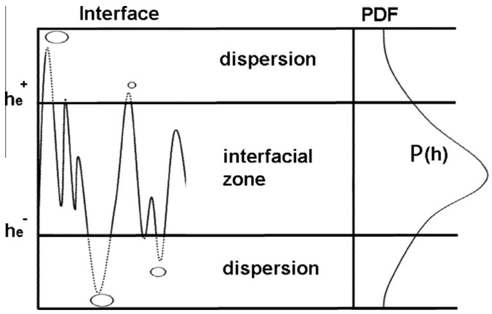

Fig. 1. The fluctuating interface is described by a probability distribution function (PDF) given by an analogy to the Boltzmann-distribution that depends solely on the given energy ratio. The interfacial energy is defined in terms of a single connected interface in the interfacial zone (the mixing zone), and we neglect a possible contribution from dispersed fluid here (no bubbles or droplets). The heights h_e^+ and h_e^- are the characteristic levels at which the stretched fluids filaments break off into droplets or bubbles.