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Error analysis in ore particle composition distribution measurements

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ABSTRACT

Measurements of ore particle composition distribution, commonly termed mineral liberation distribution, are used in assessing process performance in mineral processing. In many applications, comparisons are made between particle composition distributions (for example comparing the products of fine and coarse grinds) and in such comparisons it is useful to understand the errors in the measurements in order to decide whether any differences are significant. A statistical approach based on bootstrap resampling has been applied to estimate the confidence intervals for ore particle composition distribution measurements obtained using the MLA automated mineralogy system.

In this approach confidence intervals for each individual composition class are estimated as compared to a previous analytical solution which provides this information for particle composition data in cumulative form (Leigh et al., 1993). The effects on the magnitude of the error associated with measured values of particle composition distribution of the number of ore particles measured in the analysis and the complexity of the particle texture are investigated. Examples from a gold-bearing pyrite ore and an iron oxide copper gold ore are presented to demonstrate the practical application of this approach.

1. Introduction

Information on ore particle composition distribution, also referred to as mineral liberation distribution, is an important parameter used in processing plant design and optimisation. One example application is the determination of grinding targets for separation processes. A common approach used to collect this information and other textural features of the ores is through measurements on polished sections of particles. With the increased use of automated mineralogy systems, rapid and large amounts of measurements of mineralogical data can be conducted. The availability of these systems means that statistically reliable estimates of mineralogical data should be achievable (Sutherland and Gottlieb, 1991). While correct sampling procedures are used to minimise the error due to physical sub-sampling users of mineralogical data should also consider the measurement error which is a function of the number of particles measured and the variability of particle composition in the sample.

In practice, comparisons are often made between particle composition distributions, a common example being the comparison between the particle composition distributions of the products of fine and coarse grinds when seeking to identify the optimum grind size for an ore. When comparisons are being made it is useful to quantify the errors in the measurements in order to determine whether any differences are significant.

A statistical method to estimate the automated mineralogy systems measurement errors associated with the liberation data proposed by Leigh et al. (1993) has been used by previous researchers Wang et al. (2012) and Vizcarra (2010). This analytical solution utilizes the relationship shown in Eq. (1) to estimate the measurement variance of a mineral based on the number of mineral particles which contain more than and less than the nominated grade of the mineral of interest and the cumulative liberation value at the nominated particle composition. Composition (C) is the amount of the phase of interest present in the particle.

$$var(\widehat{Y}) \approx 0.31Y(1-Y)\left(\left\{\frac{1}{N_0}\right\}+\left\{\frac{1}{N_1}\right\}\right) \quad (1)$$

where

Y = Cumulative liberation expressed as a fraction ($0 \leqslant y \leqslant 1$) at composition C.
$\hat{Y}$ = Arcsin($\sqrt{Y}$).
N_0 = Total number of locked particles with composition less than C.
N_1 = Total number of particles with composition at least C.

Note that this analytical method described by Leigh et al. (1993) requires the liberation data to be expressed in a cumulative

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liberation yield format and does not provide a method to estimate confidence intervals for individual composition classes. Leigh and his co-workers mention that the bootstrap method is applicable to give a definitive answer with regard to confidence limits of liberation measurement; however the method to achieve this is not discussed further in the paper. The bootstrap resampling approach has been applied previously by Evans and Napier-Munn (2013) who demonstrated the applicability of a statistical methodology based on bootstrap resampling to estimate error in measurements of textural characteristics specifically on mineral grain size distribution and mineral assays as quantified by automated mineralogy systems.

Lamberg and Vianna (2007) also proposed a statistical method based on Poisson distribution to determine the relationship between coefficient of variation of the particle class in terms of mass proportion of the mineral and number of particles. The error model is shown in Eq. (2). Confidence intervals for each individual composition class are estimated using this approach.

$$CV\% = \frac{100}{N^{0.5}} \tag{2}$$

where

CV = Coefficient of variation.

N = Number of particles in the particular class.

This paper details the application of a statistical approach based on bootstrap resampling to estimate the confidence intervals for ore particle composition distribution measurements obtained using data generated by the MLA automated mineralogy system. The advantages of using bootstrap resampling approach include requiring minimal assumptions for validity and not depending on an analytical solution to investigate complex systems (Napier-Munn, 2014). In contrast to the approach taken by Lamberg and Vianna (2007) and the analytical approach taken by Leigh et al. (1993). The effects of the number of ore particles measured in the analysis and the complexity of the particle texture on the magnitude of the error associated with the measured values of particle composition distribution are also investigated.

2. Materials and methods

2.1. Sample preparation

The two ore samples used in the development of this method were a gold-bearing pyrite ore and an iron oxide copper gold (IOCG) ore. The gold-bearing pyrite ore samples used in this work consisted of size fractions of broken ore specifically the −1.18 mm+600 μm, −600+300 μm, −300+150 μm, −150+75 μm and −75+38 μm fractions. For the iron oxide copper gold ore, the samples used include Flotation Feed (FF), Rougher Concentrate (RC) and Rougher Tail (RT) each with size fractions of −300+150 μm, −75+38 μm and −28+10 μm. A number of subsamples were split from the full population of each size fraction to be submitted for textural characterisation using the FEI Mineral Liberation Analyser (MLA). Each size fraction subsample for mineralogical characterisation was mounted in polished blocks for analysis.

2.2. Mineral liberation measurements

The MLA XBSE measurement mode was used to measure the polished blocks. This mode uses backscatter electron images to delineate particles and mineral grains followed by acquisition of characteristic X-rays to identify the mineral present in each grain (Gu, 2003). The quantitative data from MLA analysis of each block were generated using the MLA Dataview software. The mineralogical data for each particle which are required as inputs to the bootstrap resampling analysis are located in the Particle Properties data table in the MLA Dataview software. The example of the Particle Properties table shown in Table 1 highlights the data which are required to estimate the confidence intervals for the liberation data, namely for each particle the mineral composition of the particle (in this case Pyrite Wt%) and the proportion of the total particle mass which the particle represents (Particle Info Wt%).

2.3. Bootstrap resampling methodology

The bootstrap resampling methodology for estimating the confidence intervals for ore particle composition distribution measurements follows a standard approach to bootstrap resampling with replacement (Efron, 1987). The method takes the original total population of measured particles in each size fraction and randomly samples N particles from this population in a bootstrap resampling with replacement approach (i.e. the sampled value is returned to the population before the next sampling is conducted). This random sampling process is repeated M times to generate M subsets of particle composition distribution data. Based on work by Efron (1987) a total of 1000 subsets were generated from the original particle set to obtain the bootstrap confidence interval (i.e. M = 1000). A computer program in MATLAB linked to a Microsoft Excel workbook has been created to execute the resampling with replacement method, wherein the original population is always considered in drawing random samples.

The inputs for the bootstrap resampling approach developed in this work are particle data from the Particle Properties table of each size fraction measured, as described in the previous section. For each subset of N particles from the resampling procedure, the data from the Particle Properties table are used to calculate the particle composition distribution. As is common practice with such data, the particle composition distribution is expressed as distribution of mineral of interest across particle composition classes and particle size fractions. The number and range of the particle composition classes and their individual ranges are set by the user.

For each particle composition class within each particle size, the mean value of the mineral proportion in that class and its standard deviation are calculated across the 1000 subsets. The coefficient of variation (COV) is calculated to enable comparison of particle composition distribution results for samples which have different mean values of the characteristic of interest.

The user is not restricted to performing the calculation for the measured number of particles but can also run the bootstrap resampling with any chosen value of the number of particles N. An alternative application described by Evans and Napier-Munn

Table 1
Example of data from the Particle Properties table required to estimate confidence intervals for the liberation data.

Particle ID	Pyrite (Wt%)	Particle information
1	100	0.09
2	100	0.47
3	100	0.01
4	99	0
5	97	0.13
6	96	0.03
7	74	0.15
8	69	0.12
9	69	0.14
10	67	0.01
11	59	0
12	55	0.22
13	53	0.2
14	52	0.1

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