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An energy model on particle detachment in the turbulent field

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ABSTRACT

A flotation detachment model is developed by considering energy balance in the process. Energies concerned are surface energy increment and kinetic energy supplied by turbulent liquid motion. Surface energy increment is the work of adhesion by surface forces which is reflected by surface tension and contact angle. What makes this model outstanding from other detachment models of energy balance perspective is more accurate account of kinetic energy supplied from turbulent liquid motion. Eddies in the same scale as attached particles are considered accountable for particle detachment in the close vicinity. In this way, detachment probability is written as a function of energy dissipation rate. Predictions from different models are compared to experimental results. It is demonstrated that previous models overestimate the influence from turbulent liquid motion. Notably, with more accurate account of eddies' influence, the new model predicts particle detachment in accordance with experimental results.

1. Introduction

Particle size is an important parameter in flotation and its influence on rate of recovery has been investigated by many researchers (Gaudin et al., 1931; Woodburn et al., 1971; Dobby and Finch, 1987; Crawford and Ralston, 1988; Jameson, 2012; Awatey et al., 2013). It is widely accepted that flotation achieves highest recovery efficiency treating mineral particles with size in the range of 20–150 µm. For larger particles, detachment is the limiting factor for low flotation rate. Reasons behind attached particles' detachment in the turbulent field were explored. Welsby et al. (2010) made careful on-site measurement of rate constant for galena flotation, and the experimental data was analyzed with reference to particle size and surface liberation. Afterwards, Muganda et al. (2011) measured the effect of contact angle on flotation rate constants based on a size-by-size basis. Based on reported experimental data above, Jameson (2012) found particle size influence on flotation recovery changing in the similar trend for particles of different liberation. The phenomenon of poor recovery for coarse particle has nothing to do with poor liberation, since even the fully liberated particles are affected in the same way with changes in the particle size. Thus, it is concluded that hydrodynamic environment in flotation cells leads to the decline in flotation recovery of coarse particles.

Schulze (1982) postulated that it was the centrifugal force that detaches particles from bubbles based on the hypothesis that particle bubble aggregate was trapped in the centre of a rotating eddy and attached particles followed liquid velocity of the eddy. Particle bubble detachment was believed to be caused by eddy where particle bubble aggregates experienced centrifugal field. Centrifugal force expression is given for isotropic turbulence applying Kolmogorov theory on turbulence. Centrifugal force exerted on the particle can be described as

$$F_a = \frac{4\pi R_p^3 \rho_p b_m}{3} \tag{1}$$

where b_m is the eddy turbulent acceleration. It can be determined by the root mean square of the difference between turbulent velocities, $\bar{u}_l$, over the distance r.

$$b_m = \frac{\bar{u}_l^2}{r} \tag{2}$$

Particle bubble aggregates are considered to be in the range of inertial sub-range. The mean velocity fluctuation of eddy in the inertial sub-range is drawn from the Kolmogorov theory (1941) of isotropic turbulence, fluctuating velocity can be expressed as

$$\bar{u}_l = c_1 (\varepsilon l)^{1/3} \tag{3}$$

c_1 is a constant equal to 1.37 and the particle rotating at a distance l from the axis, ε is kinetic energy dissipation rate per unit mass. Schulze (1982) assumed that particles moved with the same

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Nomenclature

E_1	energy barrier
E'_k	detachment energy
$E(k,t)$	turbulent energy
F_a	centrifugal force
R_{imp}	radius of impeller
R_p	particle radius
U_D	impeller tip velocity
W_a	Work of adhesion
b_m	eddy turbulent acceleration
c_1	turbulent constant in fluctuating velocity equation, Eq. (3)
c_2	turbulent constant in fluctuating velocity equation, Eq. (19)
d_B	bubble diameter
d_P	particle diameter
g	gravity acceleration
l	particle rotating distance from the axis of an eddy
m_p	particle mass
m_b	bubble mass
r	radius of rotation
p_1	pressure acting on the particle at the bottom
$\bar{u}_l$	liquid fluctuating velocity

Greek letters

ω	rotational speed
σ	surface tension
α	central angle
θ	contact angle
θ_0	contact angle of attached particles covering bubble
ρ_l	liquid density
ρ_P	particle density
ε	energy dissipation rate
κ	wavenumber
λ	eddy size
ν	dynamic viscosity
$\varnothing$	area of contact between particle and bubble

velocity as the eddy, and the radius of rotation can be substituted by bubble diameter, d_B. Then, centrifugal acceleration, described by b_m, can be given as

$$b_m = 1.9\varepsilon^{2/3}/d_B^{1/3} \tag{4}$$

Crowe et al. (1995) studied the interactions of particles with eddies. Particles and droplets with density higher than the continuous phase tended to migrate from the centre of eddy to the ridges of flow structure. On the contrary, bubbles tended to gather in the centre of flow structures. Different distribution patterns of particles and bubbles determine the movement of particle bubble aggregates in the turbulent field. When confronting with eddies, particle bubble aggregate would neither follow particle nor bubble movement. In the process of interaction, bubbles tend to flocculate in the centre of the eddy and particles tend to get away from the centre. The contradiction would just pull the attached particles off. However, there are underlined shortcomings embedded in Schulze's theory. The theory is based on the assumption that particle bubble aggregate is trapped in the eddy and particle rotate around the bubble with the same velocity as the eddy as is shown in Fig. 1. Eddy size remains dubious and influence from the presence of bubble on eddy's flow field is neglected. Moreover, inertial effect of the attached particle is overlooked. Besides, eddies that are influential are considered to be in the inertial range and corresponding lifetime of the eddy are considerably short, usually in milliseconds. It would be most likely that eddy will dissipate before attached particles can respond correspondingly.

It is commonly recognized that interactions between bubbles and eddies are dependent on relative sizes of bubbles and eddies. If bubbles are small relative to the eddies, they tend to be captured and entrained in the eddies. Bubbles with attached particles would follow the streamlines of the eddy. However, if eddies are in the same scale with bubbles, a single eddy cannot fully engulf the bubble and will act on part of its surface. Thus, smaller eddies will directly act upon the attached particles. Keeping a view on the progress of model development on predicting particle bubble detachment, it is recognized that turbulence parameters are loosely connected with attached particles' performance on bubble surface from force balance analysis. Kinetic energy is mostly conserved in the large scale eddies, while small eddies in the dissipation range transform liquid kinetic energy into other forms. When these small eddies act on the attached particles, energy is supplied for particles' detachment. It is still mysterious in the way this process occurs, but still proactive effort is needed in describing particle detachment process from first principles. It is worthwhile to analyze particle detachment process from the perspective of energy balance. The objective of this work is to develop a model in predicting particle detachment from energy perspective.

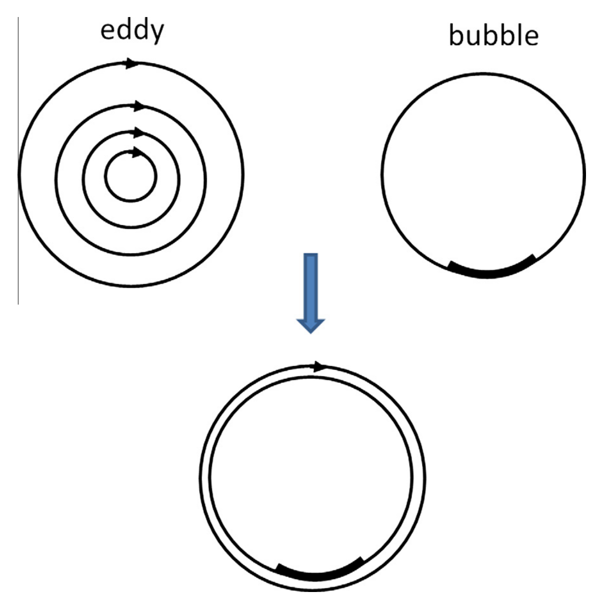

Fig. 1. Interactions between bubble and eddy.

2. Theoretical background

Yoon and Mao (1996) gave probability of detachment from the perspective of energy, where detachment probability was considered to be an exponential function of energy ratio. Energy considered is energy supplied to the attached particle and energy required for detachment to occur. In the process of detachment, two kinds of energy need to be overcome, i.e. work of adhesion and energy barrier. A particle can be detached when kinetic energy that tears the particle off the bubble surface exceeds the energy