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Dynamics of a liquid film sheared by a co-flowing turbulent gas

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ABSTRACT

Consider the dynamics of a thin laminar liquid film flowing over an inclined wall in the presence of a co-flowing turbulent gas. The solution to the full two-phase flow problem poses substantial technical difficulties. However, by making appropriate assumptions, the solution process can be simplified and can provide valuable insights. The assumptions allow us to solve the gas and liquid problems independently. Solving for the gas flow reduces to finding perturbations to pressure and tangential stresses at the interface, influencing the liquid problem through the boundary conditions. We analyze the effect of gas flow on the liquid problem by developing an integral-boundary-layer model, which is valid up to moderate liquid Reynolds numbers. We seek solitary-wave solutions of this model under the influence of gas flow via a pseudo-arclength continuation method. Our computations demonstrate that as a general trend, the wave speed increases with increasing the gas shear and the liquid flow rate. Further insight into the problem is provided via time-dependent computations of the integral-boundary-layer model.

1. Introduction

Gas–liquid flows are ubiquitous in nature, such as the shearing of the sea- and lake-water by the air flowing over it. They are also central in a wide spectrum of engineering applications. For instance, in chemical engineering, gas–liquid flows are commonly observed in the transport of hydrocarbons through long distance pipelines, absorption and distillation processes, and in a wide spectrum of processes and devices. The associated rate of heat and mass transport is significantly influenced by the hydrodynamic and physico-chemical phenomena occurring on the interfaces, hence the study of interfacial effects becomes crucial for practical applications. An accurate description of the gas–liquid interface, that in turn depends on the liquid and gas flow structure, is also essential from the fundamental point of view to, e.g. understand the various transitions occurring in the wave formation process associated with the destabilization of the interface.

The flow of a thin liquid film on an inclined plate has been the subject of many studies for a long time since the pioneering experiments of Kapitza (1965). This seemingly simple physical system can exhibit a rich dynamical behavior starting from a laminar initial film, a sequence of wave families on the surface of the film, such as periodic traveling waves and solitary waves, and eventually spatiotemporal chaos through a series of bifurcations between the different wave families. For small to moderate values of the Reynolds number, the free surface is essentially two-dimensional, comprising of solitary waves which are formed as a result of the primary wave field undergoing a secondary instability. These solitary waves have been observed in many experimental (Liu and Gollub, 1994; Vlachogiannis and Bontozoglou, 2001) and theoretical studies (Chang et al., 1995; Malamataris and Balakotaiah, 2008). Extensive reviews of falling liquid film studies can be found in the monographs by Alekseenko et al. (1994), Chang and Demekhin (2002), and Kalliadasis et al. (2012).

One of the earliest works on co-current gas–liquid film flows was that of Hanratty and Engen (1957), where the interaction between a co-flowing turbulent air stream and a thin water film was investigated experimentally. These authors reported the transition from a smooth surface to two-dimensional waves, and further to "pebbled" surfaces. Craik (1966) studied experimentally thin water films of thickness 0.13–1.6 mm in a horizontal rectangular channel, and reported the presence of fast and slow waves, that traveled faster and slower than the interface, respectively. Cohen and Hanratty (1965), through a combination of theory and experiments, analyzed a co-flowing glycerine–water solution with air in a horizontal channel. They had made use of the model of Miles (1957) and Benjamin (1959) to calculate the shear stress and pressure variations from their experimental data, and reported that the interfacial waves tend to decay, when the rate of energy transfer from the gas to the liquid is smaller than the viscous dissipation taking place within the liquid. Woodmansee and Hanratty (1969) attempted to connect the droplet entrainment from a liquid film in a horizontal co-current setting with the appearance of roll

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waves, which is typically marked by a sudden increase in the thickness of the liquid layer. Moreover, according to these authors, it is the imbalance in the pressure variations in the gas phase flowing over the wavelets, and the stabilizing effect of gravity and surface tension that leads to the entrainment of droplets from the liquid film.

Lioumbas et al. (2005) investigated the stratified co-current gas–liquid film flow in an inclined channel by experiments. The experiments were conducted at high liquid Reynolds numbers and for small inclination angles. They observed that for a given gas flow rate, small amplitude waves that exist at relatively low liquid Reynolds numbers transform to solitary waves as the Reynolds number is increased beyond a critical value. However, for large gas and liquid velocities, roll waves having a large amplitude and moving at high speed, start to appear. In their subsequent studies, Lioumbas et al. (2006, 2009) analyzed the influence of surfactants on the wave characteristics of both the upflow and downflow configurations in inclined channels. In a recent experimental study, Alekseenko et al. (2009) investigated both the entrainment and no-entrainment regimes of gas–liquid annular flows, and attempted to link the occurrence of entrainment phenomena, with the disappearance of ripples that were previously formed on the backward slope of disturbance waves.

Theoretical studies of co-current gas–liquid film flows are limited. Jurman and McCready (1989) investigated theoretically and experimentally waves on thin liquid films sheared by a turbulent gas in a horizontal setting. In their analysis a glycerine–water solution system, they reported the existence of solitary waves that travel faster than the periodic waves, when the gas Reynolds number is sufficiently large till a critical liquid Reynolds number is reached. They supplemented their experimental analysis by deriving a weakly nonlinear model using boundary-layer-type approximations, and examined the influence of dynamic and kinematic processes on the wave behavior. In their predominantly experimental study on a co-current gas sheared liquid flow problem in a horizontal setting, Peng et al. (1991) looked at the wave field both for low liquid Reynolds numbers, where solitary waves exist, and high Reynolds numbers, where solitary waves are absent. They conjectured that solitary waves originate from waves that have sufficiently large amplitude to substrate depth ratios, through a secondary transition. More recently, Frank (2006) demonstrated numerically by using the method of particles, the existence of solitary waves in a shear driven thin film flow in a horizontal channel in the presence of laminar gas flow.

The present study builds on the methodology developed by Tseluiko and Kalliadasis (2011)(referred to as TK in the following) to analyze flooding in a countercurrent gas–liquid film flow. We concentrate here on the influence of gas shear on the structure and speed of solitary waves with a gas co-flowing over a thin liquid film in a vertical channel. As part of the analysis presented here parallels the work of TK, when necessary the reader will be referred to this study for further details. For the co-current problem considered here, we solve the gas and liquid problems separately by making appropriate assumptions as in the study of TK. The gas problem in particular, is analyzed with an improved version of the quasi-laminarity approach of Miles (1957) and Benjamin (1959), which is also used by Demekhin (1981) and Trifonov (2010) but on a Cartesian coordinate system. Like in TK, we work on curvilinear boundary layer coordinates, and find perturbations to pressure and tangential stresses at the interface due to the turbulent gas flow, which are used as boundary conditions in the solution to the liquid flow problem. Instead of solving the full Navier–Stokes equation for the liquid problem, as done by Trifonov (2010), we develop an integral-boundary-layer (IBL) model, that has the advantage of being amenable to mathematical and numerical analysis for moderate Reynolds numbers. In particular, the integral-boundary-layer model of Ruyer-Quil and Manneville (1998, 2000) for free-falling liquid films, obtained by combining the long-wave approximation with a polynomial expansion for the velocity field, with the integral-boundary-layer approximation and the method of weighted residuals, is known to describe nonlinear waves sufficiently far from criticality. This approach was further extended to falling film problems with additional complexities like thermocapillary Marangoni effects (Kalliadasis et al., 2003; Scheid et al., 2005a; Scheid et al., 2005b), solutocapillary Marangoni effects induced by chemical reactions (Trevelyan and Kalliadasis, 2004; Trevelyan et al., 2012) and insoluble surfactants (Pereira and Kalliadasis, 2008).

Our study is structured as follows. In Section 2 we describe the problem set up and in Section 3 we solve the gas problem. An IBL model is developed in Section 4, which is also used to perform a linear stability analysis of the flat film solution in contact with the gas flow. Using the IBL model we discuss solitary-wave solutions in Section 5 through a continuation approach, and time-dependent computations in Section 6. Finally, we summarize our findings in Section 7.

2. Problem setting

We consider a thin liquid film flowing down a smooth solid plate under the action of gravity as shown in Fig. 1. Let θ denote the inclination angle of the solid plate with respect to the horizontal, and ρ_ℓ and μ_ℓ the density and viscosity of the liquid, respectively. A gas of density ρ_g and viscosity μ_g, confined between the gas–liquid interface below and a planar solid wall at the top, flows in the downward direction (say, as a result of being pumped from the top). We take the width of the channel occupied by the gas to be much larger than the liquid film thickness, assuming also that the gas flows much faster than the liquid, hence the gas is taken to be turbulent whereas the liquid is taken to be laminar. Let the velocity and pressure in the liquid side be $\tilde{\mathbf{u}}$ and $\tilde{p}$, respectively, and the mean velocity and pressure in the gas side be $\widetilde{\mathbf{U}}$ and $\widetilde{P}$, respectively. We assume that both the liquid and the mean gas flow are two-dimensional so that there are no variations in the transverse direction. Let $(\tilde{x}, \tilde{y})$ be a Cartesian coordinate system

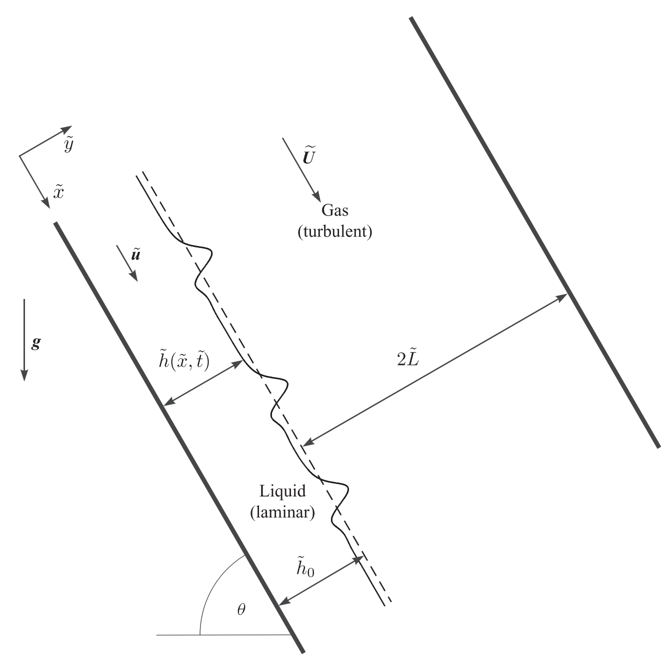

Fig. 1. Schematic of the co-current gas liquid flow problem in an inclined channel.