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A Lagrangian three-phase slug tracking framework

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ABSTRACT

A computational framework for dynamic three-phase pipe flow is presented and compared with some severe liquid slugging experiments with air–oil–water flow in an S-shaped riser. The model is based on a hybrid slug capturing and slug tracking formulation, in which a full two-fluid formulation is used in the stratified flow regions. A tracking scheme based on a single liquid momentum balance over the length of the liquid slug is used for the slug region. Slug fronts and bubble noses are tracked with a moving computational grid. This allows for correct reproduction of bubble nose velocities and eliminates numerical diffusion. A mixture liquid momentum formulation for the oil–water mixture is applied, and demonstrated with a simplistic liquid–liquid slip velocity model which includes a gradual transition from separated to mixed oil–water flow. The simulations compare well with the severe slugging experiments, both with respect to pressure amplitude and slugging period.

1. Introduction

A characteristic property of many maturing oil and gas off-shore fields is an increased water production. Water as a third phase increases the complexity of the multiphase transport problem, and hence the need for further experiments and modeling of three-phase flow systems to support both the design and operation of oil/gas production systems. Off-shore transport pipelines are characterized by their high length to diameter ratio. A common strategy is therefore to apply one-dimensional flow models.

Several codes have been specifically developed for dynamic simulation of multiphase oil and gas pipe flow systems. One of the commercial codes is OLGA, (Bendiksen et al., 1991), of which development started in the early 1980s. Early OLGA versions were based on a mixture liquid momentum formulation, later versions changed to using three momentum equations, one for each phase. OLGA utilizes the "unit cell" slug flow model, in which slug flow is treated in an averaged manner inside each computational cell. A slug tracking module is also offered, which allows for tracking of individual slugs with a separate sub-grid front tracking model. The main computational grid is stationary.

Another simulator based on the unit cell principle for hydrodynamic slug flow is LedaFlow, (Laux et al., 2008). This model is based on three momentum equations, and does in addition offer a quasi 3D model. This is a 2D model where the conservation equations are averaged for spanwise slices of a pipe geometry, allowing for calculation of velocity and holdup profiles over the pipe cross section.

An alternative to the averaged unit cell approach for slug flow is the concept of slug capturing, where individual slugs are resolved by reducing the grid length, (Issa and Kempf, 2003; Bonizzi and Issa, 2003; Bonizzi et al., 2009). This method can numerically reproduce details on the subdiameter scale, and the growth of small scale wave structures into slug flow has been demonstrated. LedaFlow is also designed to allow for slug capturing.

The following gives a description of a slug tracking scheme for three-phase oil–water–gas flows. The presented three-phase slug tracking framework is an extension of previous tracking models for two-phase gas–liquid flows, (Nydal and Banerjee, 1996; Kjølås, 2007; Renault, 2007; Leebeeck, 2010).

2. The model concept

The model has been presented for two-phase flows in (Kjeldby et al., 2013). The following is a short summary with emphasis on the extension for two liquid phases.

The computational grid is organized into borders, sections and unit objects, as shown in Fig. 1. Scalar quantities (pressure, temperature, phase fractions) are stored and computed inside sections, while velocities are computed at the borders. The computational grid is shown in Fig. 2 for the three phases oil, water and gas.

A hybrid tracking and capturing scheme is applied, where separate models can be formulated for the different types of computational objects. In the current model, the two-fluid model is solved in separated flow regions. For distributed flow regions, however, a

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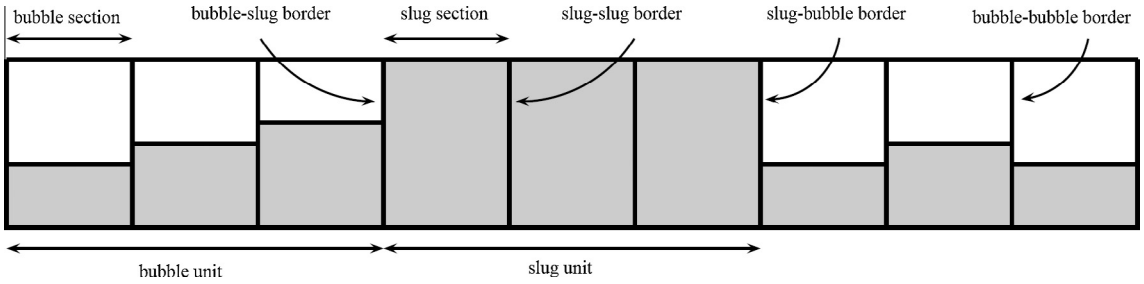

Fig. 1. Units, sections and borders.

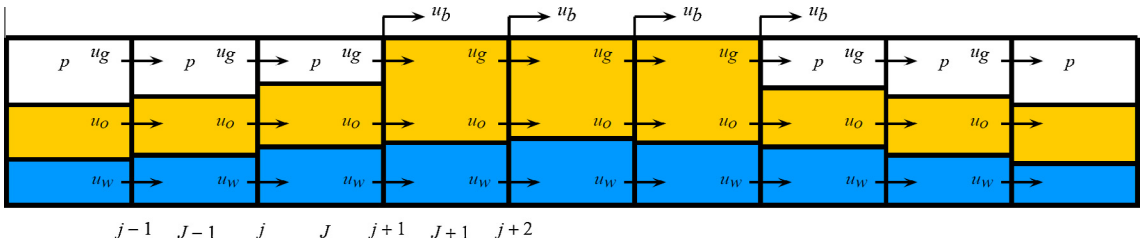

Fig. 2. The numerical grid.

single integral momentum balance is solved for each slug computational object. This reduces the problems related to increasing stiffness of the two-fluid momentum equations for single phase liquid flow.

The object oriented implementation in C++, with inheritance and with linked lists, allows for easy implementation of flow regime specific models, without the need for conditional tests. The moving grid formulation allows for non-diffusive front propagation and for implementation of front specific closure laws like Taylor bubble propagation velocity, gas entrainment at slug fronts and gas separation at slug tails. The object oriented design is suitable for multi-scale grid simulations, where some variables may be solved on a coarse grid while others may be resolved on a fine grid. The multi-scale grid also allows for slug initiation in separated flows directly from the two-fluid model on a fine grid (capturing), or by supplementary slug initiation models on a coarse grid. Once a slug is created (regardless of the initiation method), the tracking scheme with moving and deformable computational objects (moving grid) takes over.

A mixture momentum formulation is applied for the oil and water, thus requiring a closure model for oil–water slip velocity. A unique feature of the presented model is the option of simulating curved pipe geometries. Another unique characteristic is the use of a single grid system both for formulation of equations and for the slug tracking functionality.

3. Conservation equations

Integrating the mass of phase k on a moving and deformable control volume J gives the mass conservation equation for phase k:

$$\frac{d}{dt}\int_{V_{k,J}}\rho_k dV+\int_{L_J}\frac{\partial\rho_k(u_k-u_b)}{\partial x}A_k dx=\dot{M}^s_{k,J} \tag{1}$$

The subscript k is either o, w, l or g which denotes oil, water, mixture liquid or gas respectively. Here, t denotes time, ρ density, V volume, A area, L length and u velocity. A mass source of phase k is given by $\dot{M}^s_{k,J}$, while x is the pipe longitudinal axis. The border velocity is denoted u_b.

A pressure equation which is solved in bubble sections is obtained by adding the mass conservation Eq. (1) for all the phases and expanding the density from an equation of state on the following form:

$$\rho_k=\rho_k(p,T) \tag{2}$$

The pressure equation for isothermal flow then becomes:

$$\frac{\partial p_J}{\partial t}\sum_k\left(\frac{\partial\rho_{kJ}}{\partial p}\frac{V_{kJ}}{\rho_{kJ}}\right)+\frac{\partial V_J}{\partial t}+\sum_k\left(\frac{1}{\rho_{kJ}}\int_{L_J}\frac{\partial\rho_k(u_k-u_b)}{\partial x}A_k dx\right)=\sum_k\left(\frac{\dot{M}^s_{kJ}}{\rho_{kJ}}\right) \tag{3}$$

A mixture liquid momentum equation is obtained by adding the momentum equations for oil and water and integrating over the momentum control volume j:

$$\begin{aligned}\frac{d}{dt}\int_{V_{l,j}}\rho_l u_l dV+\int_{L_j}\frac{\partial\rho_l u_l(u_l-u_b)}{\partial x}A_l dx&=-\int_{V_{l,j}}\frac{\partial p}{\partial x}dV-\int_{V_{l,j}}\rho_l g\sin\theta dV-\int_{V_{o,j}}\rho_o g\cos\theta\frac{\partial h_o}{\partial x}dV\\&-\int_{V_{w,j}}\rho_w g\cos\theta\frac{\partial h_w}{\partial x}dV-\int_{L_j}\tau_l S_l dx-\int_{L_j}\tau_{lg}S_{lg}dx\\&-\int_{L_j}\frac{\partial}{\partial x}\left(\frac{\rho_o\rho_w\alpha_o\alpha_w}{\rho_l\alpha_l}Au_s^2\right)dx\end{aligned} \tag{4}$$

The momentum control volume is staggered to the mass balance control volume J. Here, the pipe inclination is denoted θ, the height of the water and the mixture liquid films h_w and h_l, the gravitational acceleration g and the volume fraction of phase k, α_k. Wall and interface shear stresses are given by τ_k and τ_{kn}, where n denotes the neighboring phase. Similarly, wall and interface perimeters are denoted S_k and S_{kn}. The oil–water slip velocity u_s is defined as:

$$u_s=u_o-u_w \tag{5}$$

The mixture liquid, oil and water phase fractions are related to each other as follows,

$$\alpha_l=\alpha_o+\alpha_w \tag{6}$$

while the mixture liquid density is calculated by: