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Rapid determination of the magnetization state of elliptic cross-section matrices for high gradient magnetic separation

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ABSTRACT

Our previous studies showed that the performance of the matrices varied greatly before and after reaching magnetization saturation and we expanded the particle capture models of circular and elliptic matrices in high gradient magnetic separation (HGMS), considering both the case that the matrices were unsaturated and saturated. Problem remained that at which condition the matrices will reach magnetization saturation in HGMS. A method to determine the magnetization state of the matrices for selecting the applicable particle capture models (models for unsaturated or saturated matrices) in specific studies should be developed. This is particularly essential for studying the effect of matrix shape (aspect ratio) on the particle capture performance of matrices in HGMS, as the magnetization sate of the matrices varies greatly with the aspect ratio. In the present paper, the magnetization process of elliptic matrices was investigated and a rapid and convenient method of judging the demarcation for selecting the applicable particle capture models was proposed and was validated with numerical simulation. Generalized relation between magnetization coefficients and the matrix shape coefficient γ (ratio of axis along the magnetization direction to that perpendicular to the magnetization direction) before and after reaching magnetization saturation were quantitatively presented. The demarcation for judging the magnetization sate of elliptic matrices in HGMS can be determined by the simple equation $B_0 = M_s/(\gamma + 1)$. Based on the equation, the magnetization state of the matrix can be rapidly determined and the applicable particle capture models can be selected accordingly in specific studies.

1. Introduction

High gradient magnetic separation (HGMS) has been widely applied in many scientific and industrial fields for the recovery of particulate matters [1–6]. Magnetic matrices are the key component of the HGMS system, playing a decisive role for the performance of the system. The most commonly applied matrices in HGMS are circular cylinders of high susceptibility. There had been many literatures investigating the particle capture of the circular cross-section matrices in HGMS and many capture models were established [7–13]. However, nearly all these models were based on the premise that the matrices were saturated by the applied magnetic field. Little literatures concerned the magnetization sate of the matrices in HGMS. Our previous studies showed that special cross-section matrices such as elliptic cross-section matrices had better magnetic characteristics than circular cylinders and consequently presented better performance in HGMS [14]. But the experimental results could not be explained with the classical particle capture models (basing on that the matrices were saturated by the applied field) proposed by previous researchers [15–17]. Through a great deal of theoretical and experimental studies, we found that the magnetization state of the matrices in HGMS was an important factor influencing the performance of the HGMS system. The performance of the matrices varied greatly before and after reaching magnetization saturation. So we expanded the particle capture models of circular and elliptic cross-section matrices (circular and elliptic matrices for short hereafter), considering both the case that the matrices were saturated and unsaturated by the applied magnetic field. Theoretical analyses with the expanded particle capture models agreed well with the experimental results in the transversal and axial configurations of HGMS [18, 19].

Although the particle capture models of circular and elliptic matrices in HGMS had been established for both the case that the matrices were unsaturated and saturated, a critical issue was that when the matrices reached magnetization saturation or how to select the right particle capture models (models for unsaturated or saturated matrices) in specific studies. In our previous papers, we had directly declared that the circular and elliptic matrices reached saturation at certain magnetic induction [18, 19], but no explanations or judging methods were provided. For elliptic matrices of the same shape (same ratio of long axis to short axis), the magnetization sate varies with the applied magnetic

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induction. Similarly, under a constant magnetic induction, the magnetization sate of the matrix will vary with matrix shape. Elliptic matrix of large ratio of long axis to short axis is prone to reach magnetization saturation under a low magnetic field. This is a vital factor which should be considered in the investigation of the effect of the matrix shape (aspect ratio) on the performance of matrices in HGMS. Elliptic matrices had present good performance in HGMS, it is essential to investigation of the effect of the matrix shape (aspect ratio) on the matrices' performance to determine whether there exists the optimal aspect ratio (this is a part of our future work). The magnetization sate of the matrices varies with the aspect ratio under a constant magnetic induction, for choosing the right particle capture models, it is required to know the magnetic state of matrices of different aspect ratios. There are many methods of measuring magnetism [20, 21], but only the basic magnetic properties of the material such as the susceptibility or the B—H curve can be obtained. In HGMS, the applied magnetic induction is a key adjustable parameter. No literature focusing on the relation between the magnetization state of a sample and the given magnetic induction. Although these relations can be investigated through finite element analyses (numerical simulation) using the B—H curve, that involves a great deal of simulation work.

So it is remarkably important to develop a rapid method to determine the magnetization state of the matrices for selecting the right particle capture models (models for unsaturated and saturated matrices) in specific studies. We had studied the demarcation of the applied magnetic induction for determining the magnetization state of circular matrix in HGMS in a previous paper (to be published). In this paper, the magnetization process of elliptic matrices in HGMS was investigated and the methods of determining the demarcation for choosing the applicable particle capture models of elliptic matrices in HGMS were proposed.

2. Magnetization principles and derivation of the magnetic field

2.1. Basic magnetization principles of matrix in magnetic field

For magnetic matrix magnetized in a magnetic field, the basic magnetization principles describe the relation among the magnetization coefficients such as the applied magnetic induction B_0, applied magnetic field H_0, the internal magnetic induction B_{in}, the internal magnetic field H_{in}, the magnetization M (saturation magnetization M_s), the susceptibility κ, the magnetic permeability μ (relative permeability μ_r) and the demagnetization field H_d. The relation among the internal magnetic induction B_{in}, internal magnetic field H_{in} and the magnetization M is as follow [22]:

$$B_{in} = \mu_0 H_{in} + M \quad (1)$$

Typically, the magnetic matrix is ferromagnetic and the magnetization M increases with increasing the applied induction B_0 before reaching magnetization saturation. When the matrix reaches saturation, the magnetization reaches the maximum M_s and then keeps constant:

$$B_{in} = \mu_0 H_{in} + M_s \quad (2)$$

The relation between the magnetization M (or saturation magnetization M_s) and the internal magnetic field:

$$M = \mu_0 \kappa H_{in} \quad (3)$$

With Eqs. (1)–(3), the following relation can be obtained:

$$B_{in} = \mu_0(\kappa + 1)H_{in} = \mu_0 \mu_r H_{in} = \mu H_{in} \quad (4)$$

where μ_0, μ_r and μ are the vacuum permeability, relative permeability and permeability of the matrix, respectively. $\mu_r = \kappa + 1$ and $\mu = \mu_0 \mu_r$. When the matrix is magnetized in an applied magnetic field H_0, a demagnetization field H_d whose direction is opposite to H_0 is generated at the matrix's ends along the direction of H_0. The relation of between H_0 and H_d is as follow [23]:

$$H_d = H_0 - H_{in} \quad (5)$$

The demagnetization field of the matrix is predominantly determined by the shape of the matrix.

2.2. Derivation of the internal and external magnetic field of elliptic matrix

2.2.1. Derivation of the magnetic field of the matrix for $\alpha = 0°$

For the determination of the magnetization state of the matrices, it is necessary to obtain the analytic expressions of the internal and external magnetic field of the matrices. The cross-section of the elliptic matrix is shown in Fig. 1. The long and short axes are $2a$ and $2b$, the focal length is $2c$. The ratio of long axis to short axis is λ ($\lambda = a/b$). The magnetic field H_0 is applied in an angle of α with respect to the positive direction of the x axis. The internal and external magnetic potentials of the elliptic matrix can be given by Eqs. (6) and (7) [24].

$$w_1 = H_0(A_1 \cos\alpha - iA_2 \sin\alpha) z_0 \quad (6)$$

$$w_2 = -\frac{1}{2} H_0 \left\{ e^{-i\alpha} \left[z_0 + \left(z_0^2 - c^2\right)^{1/2} \right] + (C_1 \cos\alpha + iC_2 \sin\alpha) \left[z_0 - \left(z_0^2 - c^2\right)^{1/2} \right] \right\} \quad (7)$$

where $z_0 = x + iy$, x and y are the coordinates in the Cartesian coordinate system, i is the imaginary symbol, e is the napierian base. A_1,A_2, C_1,C_2 are coefficients to be determined. The matrices used in HGMS are arranged regularly, so the two cases of $\alpha = 0°$ and $\alpha = 270°$ are mainly considered.

For the case of $\alpha = 0°$, substituting $\alpha = 0°$ into Eqs. (6) and (7), the internal and external magnetic potentials can be obtained. In the elliptic coordinate system, the potentials can be given by Eqs. (8) and (9).

$$\psi_1 = A_1 H_0 c \cos u \operatorname{ch} v \quad (8)$$

$$\psi_2 = -\frac{1}{2} H_0 c \cos u (e^{v} + C_1 e^{-v}) \quad (9)$$

where u and v are the coordinates in the elliptic coordinate system. At the interface of two kinds of magnetic medium, the boundary conditions are that the normal component of the magnetic induction and the tangential component of the magnetic field are continuous, as indicated by Eqs. (10) and (11).

$$B_{inv} = B_{outv} \quad (10)$$

$$H_{inu} = H_{outu} \quad (11)$$

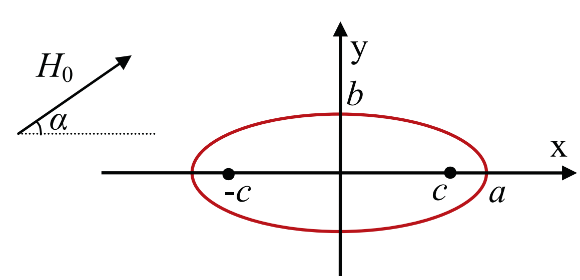

Fig. 1. The cross-section of the elliptic matrix and the applied magnetic field.