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Side-view-only determination of drag coefficient and settling velocity for non-spherical particles

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ABSTRACT

Particle settling velocity is a critical operating parameter in many industrial processes. Several correlations with reasonable accuracy exist for predicting settling velocity for non-spherical particles; however, they all share a common shortfall: they require detailed 3D knowledge of the particle shape and size that is not available in many practical cases, particularly in an on-line industrial context. These correlations are typically therefore unsuitable for use in situations where limited particle geometric information is available or where particle characteristics can change during process operation. This paper presents a method of predicting settling velocity and drag coeffecient with only 2D geometric information that can be obtained from a single side view of a particle, which is feasible to obtain using on-line imaging techniques. The correlation is formed using a large set of data obtained from the literature. We show that for a set of irregular volcanic particles not used to form the correlation, 74% of the predictions are within ±25% error. For comparison, when a standard correlation that uses 3D geometric information is applied to the same set of particles, 81% of predictions are within that error range. The results show that it is possible to obtain predictions using only side-view geometric data with an accuracy close to that achieved by other correlations that require, at minimum, particle surface area and volume. The technique works well when particles have aspect ratios less than five, a range that includes most particle shapes encountered in nature and in industrial processes. The new correlation presented here enables the prediction of settling velocity with reasonable accuracy in an on-line context for many industrial processes, which has not previously been possible.

1. Introduction

1.1. Motivation

Many industrial and environmental processes involve the settling of non-spherical solid particles in a stagnant fluid, or interaction between solid particles in a moving fluid. Examples of these processes include settling of sludge in water treatment, settling of aggregates in flocculation processes, slurry pipeline transportation, fluidized bed systems and multiphase reactors [1–3]. The rate at which a particle falls due to gravity when freely falling in an infinite medium is called the terminal settling velocity, u_∞, and it is fundamental to the models used in designing and operating many of the processes mentioned above.

Most practical applications involve solid particles settling in liquids, which is the case for most of the data considered in this work. Settling in gases is also common in large scales in atmospheric phenomena and small scales in aerosol sprays. We note that though our focus is on liquid settling data, following many past contributions and in order to show the versatility of the current approach, we do not restrict our analysis to ranges of parameters typical in liquid settling.

Several reasonably accurate correlations exist for predicting the settling rates of spherical and non-spherical particles. The most accurate and broadly cited of these are the Ganser [4] and Hölzer and Sommerfeld [5] correlations. All commonly used correlations require, at minimum, the particle's volume and a shape factor accounting for the particle shape. The most commonly used shape factor is sphericity, ϕ, which is defined as the ratio of the surface area of a sphere with the same volume as the particle and the surface area of the particle itself.[2]

Generally, at least one or two additional shape factors are also required. Some examples of these extra factors are projected-area-equivalent diameter (d_n [4]), cross-wise sphericity ($\phi_\perp$ [5]), and length-wise sphericity ($\phi_\parallel$ [5]). The common issue for all of the shape factors currently in use is that they require detailed 3D knowledge of the particles' geometry that can only be obtained by careful laboratory measurements or *a priori* knowledge of the particle shape. Even in a laboratory setting, these measurements are not trivial—and frequently not possible—for

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[2] Sphericity (ϕ) is often also denoted as ψ in the literature.

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non-spherical particles. The most obvious example of this difficulty is the measurement of particle surface area (required to calculate ϕ), which is frequently impossible for irregularly shaped, rough, or fragile particles.

In continuous industrial processes, the geometric properties of the particles often change with time. In many of these processes, it is possible to observe the particles from a side view using an on-line imaging technique, but it is not possible to remove the particles for detailed geometrical analysis. Examples of such processes include slurry transport of mineral tailings Wilson et al. [6], where the feed characteristics may vary; fluidized-bed systems Yang [7], where the solid catalyst characteristics can change through breakage or the replacement of spent catalyst; and flocculation separation processes involving breakage of fragile aggregates Vaezi G. et al. [8]. In all three cases, settling rate is the critical parameter and its prediction is a serious challenge.

At the same time, the availability of on-line side-view measurement techniques, such as Particle Vision and Measurement (PVM) [9, 10] and Focused Beam Reflectance Measurement (FBRM) [11, 12], which provide online particle characterization, is increasing rapidly. These developments provide an opportunity to predict settling rate from this type of side-view information.

In the context of this challenge and opportunity, this paper investigates two questions:

Q1. Can we replace sphericity with a side-view-accessible shape factor and still obtain a $C_D - Re$ relationship with reasonable accuracy?
Q2. Can we replace the particle volume with a side-view-accessible estimate and still obtain a prediction of u_∞ with reasonable accuracy?

We show that the answer to both of these questions is yes, and that we can now obtain reasonably accurate drag coefficient and settling rate predictions using only 2D geometric information.

1.2. Predicting terminal settling velocity, u_∞

The typical procedure to obtain u_∞ for non-spherical particles consists of three steps:

S1. Prediction of two dynamic shape factors from a particle's shape factor(s),
S2. Prediction of a $C_D - Re$ relationship from the dynamic shape factors, and
S3. Prediction of u_∞ from the $C_D - Re$ relationship.

When predicting u_∞, step S3 is performed by an iterative procedure. If the particle volume is known, the only error introduced by step S3 is a slight amplification of the error in the $C_D - Re$ relationship because Re will be estimated iteratively. This additional step S3 error is relatively small and is implicitly ignored when $C_D - Re$ correlations are presented without carrying through to a u_∞ prediction, which is frequently the case. Additionally, if the particle volume is unknown, estimating it by some other parameter (*e.g.* side-view area) will introduce a much greater error to step S3.

We note that not all approaches to determine u_∞ have exactly the form presented above. For instance, Hölzer and Sommerfeld's [5] correlation collapses the first and second steps into one with the trade-off that they require three shape factors in the calculation. Also in Ganser's [4] approach, step S1 can also be performed by experiment if the particle shape is known *a priori*. Regardless, the calculation sequence described above is suitable for describing the errors that arise using any approach and is useful for understanding the present work.

Both Ganser's [4] and Hölzer and Sommerfeld's [5] correlations demonstrate high accuracy in all three steps when the particle volume and one or two pertinent shape factors are known. (Two or three shape factors are required for non-isometric particles). This work neither criticizes nor improves on their results. Rather, we will show that using only a side-view-accessible shape factor, a $C_D - Re$ relationship with equivalent accuracy can be obtained; and much more significantly, we will show that even when the particle volume is not known and the normal shape factors are unavailable—making application of the Ganser [4] and Hölzer and Sommerfeld [5] correlations impossible—we can estimate u_∞ with reasonable accuracy.

1.3. Outline

We answer question Q1 by comparing the $C_D - Re$ relationship predicted by the present correlation to the experimental data, showing similar error to that of Ganser [4] and Hölzer and Sommerfeld [5] when d_v is known.

We answer question Q2 by comparing the settling-velocity predictions from this correlation to the measured values. Since the error level is acceptable, this method is also effective for predicting u_∞ from purely side-view accessible information.

This paper is organized as follows: Sec. 2 presents the formulation of the revised correlation using a new shape factor; Sec. 3 describes the validation of the correlation, first for the C_D calculation itself ($Re - C_D$ relationship error) and then for the final u_∞ calculation (total prediction error); next, Sec. 5 demonstrates the accuracy in predictions made for a set of u_∞ measurements that were not used in developing the correlation; and Sec. 6 summarizes the main conclusions.

2. Problem formulation

2.1. Drag coefficient and dynamic shape factors

For a freely-settling non-spherical particle, the drag coefficient (C_D) is defined as

$$C_D = \frac{4}{3}\frac{g d_v}{u_\infty^2}\frac{\left(\rho_s - \rho_f\right)}{\rho_f}, \tag{1}$$

where g is the gravitational acceleration, d_v is the diameter of a sphere with the same volume as the particle, u_∞ is the terminal settling velocity in an infinite medium, and ρ_s and ρ_f are the solid and fluid densities, respectively. The solid and fluid densities are usually known or can easily be determined. While d_v can be readily calculated for a regular geometric particle or measured for an irregularly shaped particle in a laboratory setting, in an industrial setting or for fragile particles, neither approach is likely feasible.

Ganser [4] has developed a correlation for predicting C_D based on the geometry of a given particle. His correlation uses a generalized drag coefficient, denoted C_D^*, and a generalized Reynolds number, denoted Re^*, where

$$C_D^* = C_D/K_N \text{ and } Re^* = ReK_SK_N. \tag{2}$$

In these equations, Re is the particle Reynolds number ($Re = \rho_f u_\infty d_v/\mu_f$) and K_S and K_N are dynamic shape factors that are defined as the ratios of the drag coefficients for the particle's volume-equivalent sphere and the drag coefficient for the particle itself. Specifically, K_S is this ratio in the Stokes regime ($Re < 0.1$) and K_N is the inverse of this ratio in the Newton's regime (taken as $10^4 \le Re^* \le 10^5$)[3]:

$$K_S = \left.\frac{24/Re}{C_D}\right|_{Re<0.1} \text{ and } K_N = \left.\frac{C_D}{C_{D,\text{sphere}}}\right|_{10^4 \le Re^* \le 10^5}. \tag{3}$$

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[3] We note that because of the paucity of data for disks, Ganser [4] used $10^{2.5} \le Re^*$ as the Newton's range for disks. We do the same for both disks and cylinders here except make the range $10^{2.5} \le Re^* \le 10^5$.