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The electrohydrodynamics of superimposed fluids subjected to a nonuniform transverse electric field

Asghar Esmaeeli *, Marrivada N. Reddy

Department of Mechanical Engineering & Energy Processes, Southern Illinois University, Carbondale, IL 62901, United States

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ABSTRACT

This study aims to investigate electrohydrodynamics of two superimposed fluids that are confined between a pair of two-dimensional flat plates and are exposed to a sinusoidal electric field in zero gravity. The goal is to identify the parameters that affect the flow structure and interface deformation using a simple closed form solution. The governing electrohydrodynamic equations are solved analytically for Newtonian and immiscible fluids in the framework of leaky-dielectric theory and in the limit of small electric field and fluid inertia. A detailed analysis of the electric and flow fields is presented and it is shown that the electric field induces sinusoidal electrical stresses at the interface, which lead to periodic convection cells. The parameters affecting the sense of flow circulation and strength are investigated and it is shown that the former depends on the relative magnitude of the electric permittivity and conductivity ratios while the latter is controlled by the relative thicknesses of the fluid layers and the ratio of the electric conductivities and viscosities of the fluids. The maximum flow strength is achieved at a relative thickness that is set by the competition between the electric and hydrodynamic effects. For small deformation, the distortion of the interface is examined using a normal stress balance at the interface, and it is shown that the degree of interface deformation scales with the square of the amplitude of the electric potential nonuniformity, while its wavenumber is twice that of the imposed potential nonuniformity. Furthermore, a zero-deformation curve is found, which delineates the region in the permittivity-conductivity space according to the sense of interface deformation. The results show that for certain ranges of fluid layer thicknesses and permittivity ratios, the interface will remain flat, despite the action of the nonuiform field.

1. Introduction

When a system of two immiscible fluids is exposed to a uniform electric field, the mismatch between the dielectric properties of the fluids leads to "net" normal and tangential interfacial stresses, which set the fluids in motion and tends to deform the interface. The theoretical model that describes the phenomenon fairly well is the so-called Taylor–Melcher "leaky-dielectric theory", developed concurrently by Taylor and Melcher in the context of electrohydrodynamics of drops (Taylor, 1966) and electrohydrodynamic-driven instability of superimposed fluids (see, for example, Smith and Melcher, 1967; Melcher and Schwarz, 1968; Melcher and Smith, 1969), respectively. The theoretical foundation and the mathematical formulation of the theory have been well described in the review articles by Melcher and Taylor (1969), Arp et al. (1980), and Saville (1997). The essence of the model is to assume that fluids have finite electric conductivities and that the time scale of charge relaxation due to conduction from the bulk to the surface to be much shorter than any process time of interest. The first assumption allows for the accumulation of free charges at the interface and, therefore, the possibility of net electrical shear forces at the interface; i.e., the key element that is overlooked in "electrostatic" theory, which treats the fluids as either perfect insulator or perfect conductor; see, for example, O'Konski and Thacher (1953), Basaran et al. (1989), Basaran et al. (1995), and Miksis (1981). The second assumption leads to a substantial simplification in the mathematical formulation as the electric field equations will be decoupled from the momentum equation and reduce to quasi-steady state laws.

Since its inception, the leaky-dielectric theory has been compared against many experiments, the majority of which were concerned with the steady state deformation of a solitary drop in a weak uniform electric field. Most notably one can refer to the experiments of Taylor (1966), Allan and Mason (1962), Torza et al. (1971), Vizika and Saville (1992), and Tsukada et al. (1993, 1994). The results of these studies show a perfect qualitative agreement between the theory and the experiments in predicting the sense of drop deformation and flow circulation, while some quantitative disagreements still remain between the two in the prediction of the degree of drop deformation for some cases. To improve the results, a major initiative was taken by Baygents and Saville (1989), Baygents and Saville (1991), followed up by

* Corresponding author. Tel./fax: +1 618 453 7001.
E-mail address: esmaeeli@engr.siu.edu (A. Esmaeeli).

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Zholkovskij et al. (2002) and Supeene et al. (2008), to resort to electrokinetic theory, where motion of charge is explicitly accounted for in the charge and momentum conservation equations. The picture that emerges from these studies is that the electrokinetic theory leads to the same results as those of the leaky-dielectric theory when the Debye screening length for the charge is small compared to the characteristic length scale of the problem. As such, it is generally believed that the leaky-dielectric theory is the correct ''lumped-parameter'' model when no net charge exists on the interface.

Early interest in the interaction of the electric field with fluid interfaces stemmed from deformation and break up of rain drops during thunderstorm (Macky, 1931) and the effect of deformation of aerosol on optical studies of disperse systems (O'Konski and Thacher, 1953; O'Konski and Harris, 1957). More recent interest is directed toward applications involving microfluidic systems (Stone et al., 2004; Zeng and Korsmeyer, 2004), where the surface forces become increasingly dominant as the dimensions of the device are decreased. Examples include atomization of liquid jets by electric field in continuous inkjet printing (Mutoh, 2002), manipulation of biological cells (Jayasinghe et al., 2006), coalescence of droplets for de-emulsification purposes (Mostowfi et al., 2007), electrohydrodynamic jet printing (Park et al., 2007), and electrospraying of one fluid into another (Collins et al., 2008), to name a few.

In this study, we are interested in exploring the steady-state behavior of a two-dimensional liquid bilayer due to an imposed nonuniform transverse electric field. The liquid layers are sandwiched between the walls of a horizontal channel, which also serve as the electrodes. The goal is to identify the parameters that affect the electric field-driven fluid flow and interface deformation using a simple closed form solution. To this end, we solve electrohydrodynamic equations for creeping flow of the liquids in the framework of leaky-dielectric theory, assuming a flat interface, and evaluate the interface deformation *posteriori* for small distortion from the planar shape using a domain perturbation technique. Our analytical solution builds on that of Smith and Melcher (1967) who studied a similar problem in a semi-infinite domain. The aim of these authors, however, were more limited and also they did not account for the interface deformation.

The forgoing problem finds relevance in a host of microfluidic processes involving liquid films, such as enhancement of mixing of continuous fluid streams in microchannels (ElMoctar et al., 2003; Li et al., 2007), mitigation of disruptions at the free surface in thin-film coating technologies (Tseluiko et al., 2011; Tseluiko et al., 2008a; Tseluiko et al., 2008b; Stillwagon and Larson, 1990), and creation of highly precise structures on submicron scales by destabilizing the liquid surface in soft lithography (Chou and Zhuang, 1999; Schaffer et al., 2000). The electrolithography, which is used for manufacturing of MEMS devices and semiconductors, is a particularly attractive example of the application of electric field on fluid interfaces by which the common lithographic procedures (Rizvi, 2005) is bypassed and also the technological barrier that is imposed by the wavelength of light in generation of increasingly smaller features on integrated circuits is overcome (Schwarz et al., 2004). Here, a thin polymer film is coated on a highly polished silicon wafer, serving as one of the electrode, and is placed beneath a topographically structured electrode within a short distance. An electric potential difference is then established between the two electrodes, which leads to instability of the liquid and replication of the electrode pattern. The idea of electrically induced structure formation was initiated by Chou and Zhuang (1999) and Schaffer et al. (2000), and sparked a wave of interest on the subject. See, for example, Schaffer et al. (2001), Pease and Russel (2002, 2003, 2004, 2006), Wu and Chou (2003), Shankar and Sharma (2004), Wu and Dzenis (2005), Wu and Russel (2005, 2009), Wu et al. (2005, 2006), Craster and Matar (2005) and Yeoh et al. (2007).

Of particular relevance to our work is the study by Yeoh et al. (2007), who used a geometric setup similar to ours, comprised of a liquid layer overlaid by a gas layer. Their key objectives were to determine the equilibrium shape of the interface, its instability, and the critical potential nonuniformity that leads to interface instability. They, however, considered the fluids to be inviscid and perfect dielectric. Furthermore, they did not ignore the gravitational acceleration and treated the nonuniform (sinusoidal) electric potential as a perturbation added to a uniform (base) potential. The authors performed analytical and numerical computations using the domain perturbation technique (Joseph, 1973) and Galerkin finite element method (G/FEM), respectively, and were able to determine the equilibrium shape of the interface for small and large deformations (using their analytical solution and numerical simulations, respectively) and the amplitude of the critical electric potential nonuniformity that would lead to the interface instability (using their numerical solution) as a function of the controlling parameters of their problem. What sets Yeoh et al. (2007) study (and for that matter our investigation) apart from the rest of the studies (for which a partial list was given in the preceding paragraph) is their approach toward solving the problem. As the authors pointed out, the equilibrium shape of an interface subjected to a nonuniform electric field will be nonplanar, even for field strengths below the critical values for instability, because of the variations of the electric stresses along the interface. As such, the problem cannot be solved using linear stability analysis (see, for example, Melcher, 1963) because the interface will be highly deformed even below the critical field strength. To bypass this problem, there were two main approaches in exploring the structure formation in thin liquid films by nonuniform electric fields before Yeoh et al. (2007) study. In one approach (see, for example, Thaokar and Kumaran, 2005), the insight gained from the interaction of a *uniform* electric field with a fluid interface would be used to analyze and interpret the experimental results, while in another approach (see, for example, Verma et al., 2005), the transient evolution of the phase boundary of a liquid/gas system subjected to electric field strength exceeding the critical value would be studied in the creeping flow limit and in the framework of long-wave approximation. Yeoh et al. (2007) approach, however, did not involve the long-wave approximation and also accounted for the nonuniformity of the imposed electric potential. While the scope of our work is more limited than that of Yeoh et al. (2007), as our solution is confined to small interface deformation and also we do not examine the interface instability, our analytical solution is more general than that of theirs at the same order of perturbation parameter (i.e., the amplitude of the potential nonuniformity). This is because the both analytical approaches are essentially the same, while the perfect dielectric model (used by Yeoh et al., 2007) can be treated as a special case of the leaky dielectric model (used by us). We, therefore, pay special attention to the analytical results of Yeoh et al. (2007) and compare our results in the limit of $R = S$ with their corresponding results.

The organization of this paper is as follows. Following this Section, the problem setup and the governing nondimensional numbers are discussed in Section 2. Section 3 contains the governing electrohydrodynamic equations, and in Section 4 we present the solution of the electric potential along with the interface charge distribution and electric field stresses. Solution of the streamfunction equation and the parameters that affect the strength of the convection cells and their sense of circulation are explored in Section 5. The interface deformation is evaluated in Section 6 and in Section 7 we explore the effect of separation distance between the channel walls on the results. In Section 8 we compare our results with the analytical results of Yeoh et al. (2007). Section 9 is the conclusions.