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A nonlinear packing model for multi-sized particle mixtures

Ching S. Chang *, Yibing Deng

Department of Civil and Environmental Engineering, University of Massachusetts, Amherst, MA01003, USA

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ABSTRACT

Linear packing models are commonly used to predict the specific volume of a packing mixture composed of several size classes of particles. In a linear packing model, the specific volume of a packing mixture is a linear function of the volume fractions of each size class of particles. However, predictions from linear packing model have been observed to deviate from measured experimental results of both binary and ternary mixtures. For some cases, the deviation is significant. The concept of dominant size is an important element in the packing model. In the linear packing model, the dominant size is assumed to be one of the size class of the particle mixture. However, this assumption is not suitable for mixtures of particles with certain solid volume fractions. In the present model, we recognized that the dominant skeleton is composed of more than one class of particles. To represent the state of dominant skeleton, we hypothesize a dominant size, which is a continuous variable. Using this hypothesis, a non-linear packing model is developed. The proposed model is evaluated by experimental results obtained from binary mixtures of sand-silt and ternary mixtures of steel ball bearings, steel shots, glass beads, and washed sand. Comparison of the predicted and measured results indicates that this model is able to capture the non-linear behavior of density variation with respect to solid volume fractions.

1. Introduction

Packing of particulate materials has been broadly studied for design and manufacturing in many fields, such as mineral, metallurgical, pharmaceutical, and geotechnical industries [1–4]. Two approaches have been adopted for modeling the density of particle packing: (1)computer simulation and (2)analytical method. A brief summary of these two approaches is given below.

With the rapid advances in computer technology, the method of computer simulation has been progressed significantly. Computer simulation has been applied by many researchers for studying geometric packing properties of granular material composed of a range of particle sizes [5–8] and for studying the effect of polydispersity on mechanical response of granular material [9–11]. Many research work have been devoted specifically to the geometric properties and bulk mechanical properties of binary mixtures [12–18].

The results of computer simulation resemble the experimentally measured density of particle packing. However, due to the wide range of particle sizes in real granular soil, within a specimen, the size of large particles is thousand times larger than the size of small particles. Thus the specimen required to be a representative sample of a packing mixture would contain enormous amount of particles, which is much too large to be simulated on present computers. As a consequence, the computer simulation method is impracticable to simulate soil with real particle sizes.

Moreover, as pointed out by Dodds [19], the computer simulation is just one type of experimental measurements, which provides the results of a specific mix composition. The results cannot be applied to the packing densities of other mix composition, unless with the aid of some interpretative model. On the other hand, analytical methods require some approximation and hypothesis at a scale larger than particle so that a mathematical framework can be established to efficiently solve for the packing densities for various mix compositions. Therefore, the analytical modeling is useful from the point of view of engineering application, although it may be empirical to some degree.

Development of analytical method has been pioneered by Westman and Hugill [20] and Furnas [21] for the prediction of the packing density of binary mixtures. They found that packing density is influenced primarily by two distinct particle mix mechanisms: the filling mechanism, which takes place when small particles are filled into the large-particle skeleton, and the embedment mechanism, which occurs when large particles are embedded into the small-particle matrix. However, their methods are limited to the extreme case of very large particle-size ratio (i.e. the size of large particles is substantially greater than the size of small particles).

In the sixties, considerable progress was then made to predict the density for a binary mixture of spherical particles with smaller particle-size ratios. Powers [22] considered the loosening effect in the filling mechanism: for smaller particle size ratio, the size of interstices among large particles may be smaller than the size of small particles. Thus,

* Corresponding author.
E-mail addresses: cchang@engin.umass.edu, (C.S. Chang), yibingdeng@engin.umass.edu (Y. Deng).

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when small particles are filled into the large-particle skeleton, the small particles push apart the surrounding large particles and loosen the skeleton. Aïm and Goff [23], and Toufar etal. [24], took account of the wall effect in the embedment mechanism: for smaller particle size ratio, when a large particle is embedded into the small-particle matrix, noteworthy void spaces are created at the interface between the large particle and the surrounding small particles.

After that, more versatile models were developed by Stovall etal. [25], Yu and Standish [26, 27], Goltermann etal. [28], de Larrard [29] and Dewar [30], taking account of both loosening and wall effects by using a filling-effect parameter and an embedment-effect parameter, which are functions of particle size ratio. Various functions have been proposed based on the particulate material used in various studies.

The abovementioned models were mainly developed with the concept of spherical particles and verified with spheres or round-particles. It is known that the particles used in chemical or pharmaceutical industries are usually non-spherical. Yu and Standish [31, 32] demonstrated that a direct application of the abovementioned analytical models cannot predict the results of sphere-rod mixtures from Milewski [33, 34]. Yu etal. [35] modified the analytical model for spheres to be applicable for cylinder particles. Besides the analytical models, a number of numerical simulations have been found for ellipsoidal particles [36–39] and polyhedral particles [37, 40, 41]. For binary mixtures, the experimental and numerical simulation work were to a large degree limited to the sphere-rod (or cylinder) [33, 34].

Jones etal. [42] compared the three models by Goltermann etal. [28], De Larrard [29] and Dewar [30], and found that their accuracy and applicable range of particle size ratio vary substantially. The differences between these packing models indicate that each model has limited applications for certain industry material pertaining to the particle geometry and the range of particle sizes of the specific material used. Soil, as a natural deposit, covers a large range of particle sizes and shapes. Thus, the applicability of these models to soil as a natural deposit is questionable. For this purpose, Chang etal. [43] developed a packing model for sand-silt mixtures.

For most of the existing packing models for a binary system, a common deficiency of the prediction ability is observed. Compared to the experimental results, the predictions are generally good at low fines content (<25% when the large-particle skeleton is dominant), and at high fines content (>50% when the small-particle skeleton is dominant). However, the discrepancy becomes larger when the fines content is between 25%–50%, which are near the optimum for maximum density. Often times, the discrepancies are found to be significant and unacceptable.

Yu and Standish [44] recognized this limitation of linear packing model. They believed that the linear packing theory cannot fully describe the packing mechanism. They developed a linear-mixture packing model by combining the linear packing model with a polynomial model. The use of polynomial model amends the limitation of the linear packing model. However, the model requires several additional parameters for the added polynomial model. De Larrard [29] believed that the linear packing theory should be valid if the material is sufficiently compacted. The discrepancy is caused by the fact that the particulate material is not fully compacted. Therefore, he added another parameter, *compaction index*, to consider the effect due to insufficient compaction. This modification reduces the discrepancy. On another perspective, Kwan etal. [45] believed that the discrepancy is caused by neglecting the wedging phenomenon, and developed a model with one additional parameter, which allows for wedging effect, thus reduces the discrepancy.

Instead of using additional parameters [29, 44], Chang and Deng [46] developed a particle packing model by considering a concept of effective dominant skeleton of a binary mixture. In this packing model, the discrepancy can be greatly reduced while no additional parameter is needed.

Herein, the above model by Chang and Deng [46] is extended for the application to multi-size packing. In what follows, in order to make the model development more logical, we will begin with the analysis of the linear model. Then the model will be extended to include the concept of effective dominant skeleton. Based on this postulation, we formulate a non-linear governing equation of packing void ratio considering two mechanisms of particle mixing; the filling mechanism and embedment mechanism for a packing mixture composed of particles of various sizes. Finally, the developed new model is verified for its accuracy and applicability by comparing predicted and measured results on binary and ternary mixtures. The results show that the agreement between measured and predicted results is very good for the proposed non-linear model.

2. Conventional linear packing model

In this section, the linear packing model developed by Chang etal. [47] is briefly described. Since the model by Chang etal. [47] was motivated by problems from soil mechanics of silty sand, in which, the variable of void ratio is commonly used. For convenience, void ratio, instead of packing density, is used for the description of linear packing model in this section. It is noted that packing density ϕ is uniquely related to void ratio e. The void ratio e is defined as the ratio of void volume to solid volume of a granular packing. The packing density ϕcan be related to the void ratio e by:

$$\phi = \frac{1}{1+e}; e = \frac{1}{\phi} - 1 \tag{1}$$

We consider a packing mixture composed of n size classes of particles: size class 1, size class 2, size class 3, etc. Let d_i be the size and V_{si} be the solid volume for the ith size class of particles, respectively. It is noted that the total solid volume V_s of the packing mixture is the summation of V_{si} for all size classes, thus the solid volume fraction y_i for the ith size class of particles can be computed by

$$y_i = \frac{V_{si}}{\sum V_{si}} \tag{2}$$

It is noted that the composition characteristics of a packing mixture can be described by the two sets of values of $d_i(i = 1, n)$ and y_i $(i = 1, n)$. The packing characteristics of each size class of the packing mixture can be described by another set of values e_i (i= 1, n), where e_i is the fully packed void ratio for the i^{th} size class of particles defined by

$$e_i = \frac{V_{vi}}{V_{si}} \tag{3}$$

where V_{vi} is the void volume of the fully packed i^{th} size class of particles.

The purpose of a packing model is to analytically determine the fully packed void ratio of the particle mixture e by knowing its composition and the packing characteristics of each size class of particles (i.e. d_i, y_i, and e_i, $i = 1, n$). The fully packed void ratio, e, of a packing mixture composed of n size classes of particles is defined by

$$e = \frac{V_v}{\sum V_{si}} \tag{4}$$

where V_v is the void volume of the packing mixture.

Since the composition and the packing characteristics are known, the task of the analytical model is to predict the value of void volume of V_v of the packing mixture shown in Eq.(4).

The most commonly used model for this purpose is linear particle packing model. The algorithm used in the linear particle packing model is considered as analogous to a system of solutions of thermodynamics. Consequently, the void volume of the particlemixture can be