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# Model A vs. Model B in the modelling of particle-fluid flow

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ABSTRACT

Two-fluid model (TFM) has been widely used to simulate particle-fluid flows, with two model formulations available: Model A and Model B [1–3]. Previous studies have shown that both models generate comparable results for some flows, but their possible application limitations have not been well addressed. Recently, Zhou et al. [4] discussed this issue in the framework of coupled CFD (computational fluid dynamics) and DEM (discrete element method), indicating that both models are largely applicable to simple flows such as fluidization and pneumatic conveying, but the so-called Model B is not applicable to complicated three-dimensional flows such as that in a hydrocyclone. However, it is not clear such limitations still exist in TFM. In this work, both Model A and Model B are applied to model two typical cases, i.e., gas-solid fluidized bed and hydrocyclone. It is demonstrated that Model B is not applicable to hydrocyclones while both models are applicable to fluidized beds. The results confirm that, Model B, as a simplified model, is not applicable to the flows where the pressure gradient force is significant and its direction is quite different from that of the drag force. To overcome this problem, its original formulations, which are somehow ignored in the literature, should be used.



## 1. Introduction

Two-fluid model (TFM) has been proposed to describe particle-fluid flow since the 1960s [1,2,5,6]. In TFM, both fluid and solid phases are treated as interpenetrating continuum media in a computational cell that is much larger than individual particles but still small compared with the size of process equipment. Anderson and Jackson [5] developed one of the first set of the governing equations of TFM by local-averaging the point equation of the motion of the fluid and the motion of the mass center of a single particle, called as Set I formulation in this work:

$$\frac{\partial\left(\rho_f\varepsilon_f\mathbf{u}_f\right)}{\partial t}+\nabla\cdot\left(\rho_f\varepsilon_f\mathbf{u}_f\mathbf{u}_f\right)=\nabla\cdot\boldsymbol{\xi}_f-S_{f-s}+\rho_f\varepsilon_f\mathbf{g}\quad\text{(fluid phase)}\tag{1}$$

$$\frac{\partial(\rho_s\varepsilon_s\mathbf{u}_s)}{\partial t}+\nabla\cdot(\rho_s\varepsilon_s\mathbf{u}_s\mathbf{u}_s)=n\boldsymbol{\varphi}-\nabla\cdot\mathbf{S}+S_{f-s}+\rho_s\varepsilon_s\mathbf{g}\quad\text{(solid phase)}\tag{2}$$

where subscripts $f$ and $s$ represent the fluid phase and solid phase, respectively. $\varepsilon$, $\mathbf{u}$, $t$, and $\rho$ are, respectively, the volume fraction, mean fluid velocity, time, and fluid density for either the fluid phase or the solid phase. $\boldsymbol{\xi}_f$ is the fluid stress tensor. $S_{f-s}$ is the total volumetric particle-fluid interaction force and $n$ is the number of particles per unit volume. $\boldsymbol{\varphi}$ is the local mean value of particle-particle interaction force. $\mathbf{S}$ is the tensor representing "Reynolds stresses" for the solid phase.

Anderson and Jackson further introduced some constitutive relationships to model the unclosed terms in Eqs. (1) and (2), including: (i) replacing $n\boldsymbol{\varphi}-\nabla\cdot\mathbf{S}$ with $\nabla\cdot\boldsymbol{\xi}_s$ which represents the solid stress tensor; (ii) $\boldsymbol{\xi}_f$ and $\boldsymbol{\xi}_s$ are analogous to that for the stress tensor in a Newtonian fluid, and can be expressed as $\boldsymbol{\xi}=-P\boldsymbol{\delta}_k+f(\lambda,\mu,\mathbf{u})$ where $P$ is the local mean fluid pressure, and $\lambda$ and $\mu$ are, respectively, the effective bulk and shear viscosities; (iii) decomposition of $S_{f-s}$ into two components, i.e., $S_{f-s}=\varepsilon_s\nabla\cdot\boldsymbol{\xi}_f+S'_{f-s}$, where $\varepsilon_s\nabla\cdot\boldsymbol{\xi}_f$ is due to "macroscopic" variations in the fluid stress tensor on a large scale compared with the particle spacing and mainly includes the pressure gradient force (PGF) and the viscous force. $S'_{f-s}$ due to "detailed" variations in the stress tensor induced by fluctuations in velocity as the fluid passes around individual particles and through the interstices between particles and mainly includes the drag force in the direction of the relative velocity between fluid and solid phases. $S'_{f-s}$ also includes other forces such as the virtual mass force and lift force. The resulting equations are given in the following, called as Set II formulation in this work:

$$\frac{\partial\left(\rho_f\varepsilon_f\mathbf{u}_f\right)}{\partial t}+\nabla\cdot\left(\rho_f\varepsilon_f\mathbf{u}_f\mathbf{u}_f\right)=\varepsilon_f\nabla\cdot\boldsymbol{\xi}_f-S'_{f-s}+\rho_f\varepsilon_f\mathbf{g}\quad\text{(fluid phase)}\tag{3}$$

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**Nomenclature**

| | |
|---|---|
| **f** | force, N |
| **g** | gravity acceleration vector, 9.81 m/s$^2$ |
| $n$ | number of particles in a computational cell, dimensionless |
| $P$ | pressure, Pa |
| $t$ | time, s |
| $S$ | momentum source, N/m$^3$ |
| **S** | "Reynolds stresses" tensor for solid phase, s |
| **u** | fluid velocity vector, m/s |

*Greek letters*

| | |
|---|---|
| $\beta$ | drag coefficient, dimensionless |
| $\varepsilon$ | porosity, dimensionless |
| $\lambda$ | bulk viscosity, kg/m/s |
| $\mu$ | shear viscosity, kg/m/s |
| $\eta$ | parameter defined in Eq. (12), dimensionless |
| $\rho$ | density, kg/m$^3$ |
| $\phi$ | local mean value of particle-particle interaction force, N/m$^3$ |
| $\boldsymbol{\xi}$ | stress tensor, Pa |

*Subscripts*

| | |
|---|---|
| $f$ | fluid phase |
| $f$-$s$ | between particle and fluid |

$$\frac{\partial(\rho_s\varepsilon_s\mathbf{u}_s)}{\partial t}+\nabla\cdot(\rho_s\varepsilon_s\mathbf{u}_s\mathbf{u}_s)=\varepsilon_s\nabla\cdot\boldsymbol{\xi}_f+\nabla\cdot\boldsymbol{\xi}_s+S'_{f-s}+\rho_s\varepsilon_s\mathbf{g}\quad\text{(solid phase)}\tag{4}$$

In order to eliminate the fluid stress tensor term $\nabla\cdot\boldsymbol{\xi}_f$ in Eq. (4) for solid phase, $\nabla\cdot\boldsymbol{\xi}_f$ can be solved or expressed from Eq. (3):

$$\nabla\cdot\boldsymbol{\xi}_f=\left[\frac{\partial\left(\rho_f\varepsilon_f\mathbf{u}_f\right)}{\partial t}+\nabla\cdot\left(\rho_f\varepsilon_f\mathbf{u}_f\mathbf{u}_f\right)+S'_{f-s}-\rho_f\varepsilon_f\mathbf{g}\right]/\varepsilon_f\tag{5}$$

By setting $\frac{\partial(\rho_f\varepsilon_f\mathbf{u}_f)}{\partial t}+\nabla\cdot(\rho_f\varepsilon_f\mathbf{u}_f\mathbf{u}_f)=0$ which means the fluid flow is steady and uniform, $\nabla\cdot\boldsymbol{\xi}_f$ can be approximately expressed as:

$$\nabla\cdot\boldsymbol{\xi}_f\approx S'_{f-s}/\varepsilon_f-\rho_f\mathbf{g}\tag{6}$$

Substituting Eq. (6) into Eq. (4) (to calculate the pressure gradient force and the viscous force acting by fluid on particles) leads to.

$$\frac{\partial(\rho_s\varepsilon_s\mathbf{u}_s)}{\partial t}+\nabla\cdot(\rho_s\varepsilon_s\mathbf{u}_s\mathbf{u}_s)=\frac{S'_{f-s}}{\varepsilon_f}-\rho_f\varepsilon_s\mathbf{g}+\nabla\cdot\boldsymbol{\xi}_s+\rho_s\varepsilon_s\mathbf{g}\quad\text{(solid phase)}\tag{7}$$

It should be pointed out that the assumption of steady and uniform flow in Eq. (6) is only used to estimate the PGF and viscous force which represent the forces acting on particles by fluid due to fluid stress tensor. The physical meaning of this assumption could be understood as follows. According to Eq. (5), $\nabla\cdot\boldsymbol{\xi}_f$ is contributed mainly by three parts: the acceleration of fluid flow, the source term and the fluid gravity. Eq. (6) actually suggests that the contribution of the acceleration to the pressure gradient and deviatoric stress tensor of the undisturbed fluid flow is negligible. In other words, the fluid stress tensor of the pure fluid flow without particles is zero when the gravity force is not considered. This assumption is applicable to many gas-solid flows where the pressure gradient of pure gas flow is much less than that of gas-solid flow and the viscous force is negligible. However, as demonstrated in this work, it would not applicable to the situations such as swirling liquid-solid flows where the pressure gradient of pure fluid flow is significant and not negligible.

Comparing Eqs. (2) and (7), it can be seen that the term of $\frac{S'_{f-s}}{\varepsilon_f}-\rho_f\varepsilon_s\mathbf{g}$ in Eq. (7) should be equivalent to the term of $S_{f-s}$ in Eq. (2) since they are both the total volumetric particle-fluid interaction force. Therefore:

$$S_{f-s}=\frac{S'_{f-s}}{\varepsilon_f}-\rho_f\varepsilon_s\mathbf{g}\tag{8}$$

Thus, the term of $S_{f-s}$ in Eq. (1) can be substituted by $\frac{S'_{f-s}}{\varepsilon_f}-\rho_f\varepsilon_s\mathbf{g}$, which leads to:

$$\frac{\partial\left(\rho_f\varepsilon_f\mathbf{u}_f\right)}{\partial t}+\nabla\cdot\left(\rho_f\varepsilon_f\mathbf{u}_f\mathbf{u}_f\right)=\nabla\cdot\boldsymbol{\xi}_f-\frac{S'_{f-s}}{\varepsilon_f}+\rho_f\mathbf{g}\quad\text{(fluid phase)}\tag{9}$$

Eqs. (7) and (9) are called as Set III formulation in this work where the fluid stress tensor term is not shown in the momentum balance equation of solid phase, i.e., Eq. (7). For gas-solid flows where only the drag force and PGF are considered as particle-fluid interaction forces and the buoyancy force $\rho_f\varepsilon_s\mathbf{g}$ is negligible for small ratio of fluid-to-solid density, Eq. (8) can be simplified into:

$$S_{f-s}=\frac{S'_{f-s}}{\varepsilon_f}\tag{10}$$

For the situations where the term $S'_{f-s}$ only represents the drag force (other forces such as the lift force are negligible), $S'_{f-s}$ can be expressed as $S'_{f-s}=\beta_A(\mathbf{u}_f-\mathbf{u}_s)$ and correspondingly

$$S_{f-s}=\frac{S'_{f-s}}{\varepsilon_f}=\frac{\beta_A(\mathbf{u}_f-\mathbf{u}_s)}{\varepsilon_f}=\beta_B(\mathbf{u}_f-\mathbf{u}_s)\tag{11}$$

where $\beta_A$ and $\beta_B=\beta_A/\varepsilon_f$ are referred to as the drag coefficient [2,3]. For fluidization at minimum gas velocity where the flow can be largely regarded as uniform and steady (which means that the assumption in Eq. (6) is reasonable) and the weight of gas is negligible for high solid-to-gas density ratio, Eq. (1) can be written into $\nabla\cdot\boldsymbol{\xi}_f=S_{f-s}$. If the viscous force is neglected, we have -$\nabla\cdot P_f=S_{f-s}$. As the volumetric total particle-fluid force $S_{f-s}$ should be equal to the volumetric weight of particles to suspend the particles, the pressure gradient is equal to the volumetric weight of particles. This means that the particle weight is equal to the gas pressure force in the gravitational direction in gas fluidization [3]. Note that a force is a vector, hence the relationships of Eqs. (10) and (11) or $\beta_B=\beta_A/\varepsilon_f$ are only valid when both the drag force and PGF are in the same direction. Otherwise, significant errors could be generated and this will be demonstrated in Section 3 for the case of hydrocyclone.

Notably, Set II and III formulations are widely used to investigate particle-fluid flows. They are called as Model A and Model B, respectively [1–3,7]. The difference between Model A and Model B is sometimes considered to be related to the treatment of the pressure source term in the governing equations. If the pressure is attributed to fluid phase only, it is referred to as Model B. If the pressure is shared by both the fluid and solid phases, it is referred to as Model A. Therefore, Set I formulation given by Eqs. (1) and (2) is similar to that of Model B. However, as discussed above, Model B is actually developed after introducing various assumptions to estimate the PGF and the viscous force. It is just a simplified Set I formulation. Therefore, Set I formulation is referred to as the original Model B as discussed by Zhou et al. [4]. Previous studies have shown that Model A and Model B can generate