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Determining an optimal interstitial filling condition: An Attainable Region approach

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ABSTRACT

In this article, we present yet another application of the Attainable Region (AR) method to data from a laboratory scale milling of a low grade gold ore. In this particular case, we investigate how to optimize the amount of material in a desired size class for a scenario where the boundaries of the desired size class of interest are changed. The AR approach has never been applied in such a scenario before. Using a mono sized feed of $-1700 + 850$ µm, two desired product size classes of interest ($-850 + 150$ µm and $-150 + 75$ µm) are selected in order to determine the optimum interstitial filling (U) to be used in the mill. Two different values of U, 1.75 and 1.0 respectively, are obtained as optimal. Our results also show that optimal operating conditions are different for different objective functions. We demonstrate that the AR may be used to specify optimal conditions that may be used for particle size reduction processes. We also illustrate how an AR boundary for optimization purposes can in certain cases be extended using the ‘mixing principle’.

1. Introduction

A number of researchers have studied the comminution process with the intention of establishing its optimum operating parameters. The breakage kinetics [1–3], load behavior [4,5] and mill power [6,7] have been investigated in terms of ball size, interstitial filling, speed of the mill and other variables [8]. However, very little research has been done on integrating modern day technology into specifying the optimal operating conditions of these pre-historically determined parameters. This is mainly due to the conservative nature of the mineral processing industry.

Obtaining optimal policies in chemical reactor and grinding mill operation has been a popular problem and has been given much attention by many researchers due to the great importance and similarities that lie between the fields of reaction engineering and particle size reduction in many industrial processes.

Thus, for instance what is needed is a technique that can be employed to specify the operating conditions of a comminution process e.g. ball size distribution, slurry density, power requirements and interstitial filling, in order to optimize the combined processing units, namely the comminution unit plus any subsequent mineral recovery units. Optimization of some operational parameters of the comminution circuit has traditionally been done through the use of a classifier that is used in order to separate the oversized material and return it back to the mill for regrinding purposes. The choice of the “best” classifier of course assumes that the best size specification required by the subsequent concentration processes is known. The specification of the required size (e.g. a minimum percentage passing through a screen of a certain size) has traditionally been achieved based purely on experience.

The application of the AR technique in comminution is gaining ground and popularity, however the full potential of the technique has not yet entirely been exploited. In this paper, we extend the application of the AR technique to investigate how to optimize the amount of material in a desired size class for a scenario where the boundaries of the desired size class of interest are changed. The AR approach has never been applied in such a scenario before. The work covered in Hlabangana et al. 2016 is quite different from this investigation in that the former article used a Gold ore sample from Carletonville, while the latter used a sample from the West Rand region, of South Africa. The size classes used as well as the experimental procedures employed in these two investigations are also different.

1.1. The Attainable Region technique

In 1964, Horn [9] came up with the AR idea with an intention of using it to optimize chemical reactor structures. He showed that if it was possible to compile a set of possible outputs/outcomes for all

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possible reactor systems for a given set of kinetics and feeds, finding the optimum configuration would be simple. He then called this set of outputs the AR. His idea was to determine all achievable outcomes, from all possible reactor configurations, as opposed to just searching over a concentration space from a single reactor's output. This meant that the AR was a full set of outcomes that could be achieved by all possible designs (possibly subject to some constraints) of the processing units in response to the feed conditions. However, Horn had not given a clear set of instructions on how one can construct the AR in general. This is why the AR concept developed into a wide scope, and evolved over the years.

However, this tool was not put into practice for 20 years after its development as Horn and those who followed up on the research faced challenges on how to get a systematic way of identifying the AR. Between 1987 and 1997, Glasser and coworkers [10,11] researched on implementing a geometrical method to the construction of an AR, in which the vector description of the processes of reaction and mixing were used.

The AR geometric technique was then implemented and used extensively for the optimization of reactor networks. The analysis allows us to solve process synthesis and general optimization problems by giving guidelines for constructing the AR, as well as a methodology to determine the optimum conditions or parameters.

Due to the similarities between chemical reaction and comminution, Khumalo and coworkers [12–14] applied the AR approach to optimize comminution processes. The reasoning behind was that since a mill is analogous to a reactor, in which feed particles ("reacting") are reduced to a smaller product, the AR approach can also be equally applied in this field of chemical engineering. Using different ores, several other researchers have applied the AR technique in order to optimize different milling parameters such as speed [15], media size [16], slurry density [17] and interstitial filling [18].

Fig. 1 shows an example two dimensional (2D) AR plot of the mass fraction of material in size class of interest (M2) versus that in the feed size class (M1). The AR can either be a 2D or 3D plot and is always constructed from the feed point, which is a point on Fig. 1 with (M1; M2) coordinates of (1; 0). Finding the optimum solution to any problem is then an easy task as it involves searching over the boundary of the curve, for co-ordinates, of where the linear expression of the objective function just touches the curve. Fig. 1 shows two example cases with different linear expressions of objective functions which are tangential to the boundary, at different points. The expression, S = M2 gives the maximum amount of M2 generated and a value of 0.245 is obtained. The expression $P = 600$ M2–M1 is also another objective function expressed in terms of M1 and M2. The objective function could be that of getting more of M2 but getting a penalty for producing M1. For this objective function, an optimal M2 value of 0.235 is obtained. Also, it is possible that from knowing the processes that are required to reach the optimum point, the optimal flowsheet can be determined.

The AR technique offers a great advantage in that it centers on the fundamental processes in a system rather than the equipment used. Another advantage of the AR approach lies in that if the desired space is known, it is possible to search in that region to obtain conditions that maximize the desired objective. The technique is able to simplify the optimization problem because searching over a defined space for a maximum value of an objective function is a simple and straightforward procedure. Thus if it can be readily done for a wide range of objective functions it satisfies our conditions for being an approach suitable for the overall optimization, that is at the same time choosing the best operating conditions for each of the comminution and separation equipment to optimize the overall process.

The chosen objective function value can also be used as a yardstick for purposes of comparison. Applying the AR technique requires that the fundamental processes occurring during comminution be identified. We identified these processes to be size reduction, mixing, and classification. In this article, we will focus our work on the first two processes.

1.2. Mixing

In the formulation of a graphical approach to the AR problem, Glasser et al. [11] stipulated that a set of conditions must be satisfied in order to obtain all the possible products needed to construct the AR. He listed the necessary conditions as:

- The region should have a feed point; we can attain this point and must therefore be contained with all attainable points.

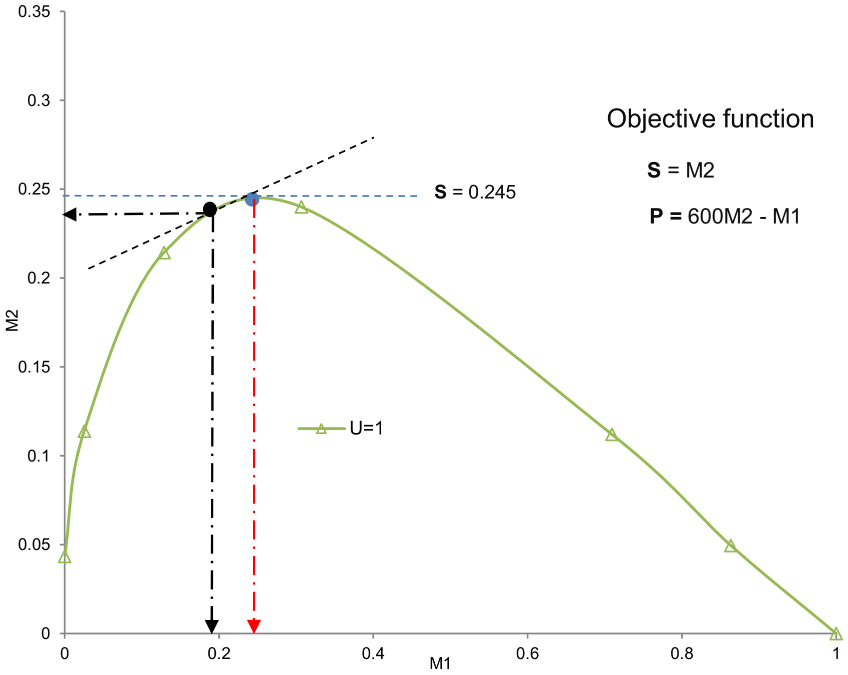

Fig. 1. Example AR plot of M2 vs M1, with different linear objective functions.