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Fiber suspension flow in a tapered channel: The effect of flow/fiber coupling

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ABSTRACT

A numerical model for predicting the flow and orientation state of semi-dilute, rigid fiber suspensions in a tapered channel is presented. The effect of the two-way flow/fiber coupling is investigated for low Reynolds number flow using the constitutive model of Shaqfeh and Fredrickson. An orientation distribution function is used to describe the local orientation state of the suspension and evolves according to a Fokker–Plank type equation. The planar orientation distribution function is determined along streamlines of the flow and is coupled with the fluid momentum equations through a fourth-order orientation tensor. The coupling term accounts for the two-way interaction and momentum exchange between the fluid and fiber phases. The fibers are free to interact through long range hydrodynamic fiber–fiber interactions which are modeled using a rotary diffusion coefficient, an approach outlined by Folgar and Tucker. Numerical predictions are made for two different orientation states at the inlet to the contraction, namely a fully random and a partially aligned fiber orientation state. Results from these numerical predictions show that the streamlines of the flow are altered and that velocity profiles change from Jeffery–Hamel, to something resembling a plug flow when the fiber phase is considered in the fluid momentum equations. This phenomenon was found when the suspension enters the channel in either a pre-aligned, or in a fully random orientation state. When the suspension enters the channel in an aligned orientation state, fiber orientation is shown to be only marginally changed when the two-way coupling is included. However, significant differences between coupled and uncoupled predictions of fiber orientation were found when the suspension enters the channel in a random orientation state. In this case, the suspension was shown to align much more quickly when the mutual coupling was accounted for and profiles of the orientation anisotropy were considerably different both qualitatively and quantitatively.

1. Introduction

In this work, we investigate the effect of the two-way coupling between the flow field and the orientation state of rigid fiber suspensions flowing through a tapered channel. Flow in the channel is governed by Cauhy's momentum equations for viscous, incompressible, planar, isothermal flow, using the constitutive model of Shaqfeh and Fredrickson (1990) to describe the local stress contribution from the fiber phase. The fiber concentration considered here is semi-dilute, which is be defined mathematically through the following relationship (e.g. Doi and Edwards, 1984):

$$1 \leqslant nL^3 \leqslant \frac{L}{d} \tag{1}$$

where n is the number density of fibers in the suspension, that is, the number of fibers per unit volume, L is the fiber length and d is the fiber diameter. In this study, we consider suspensions with identical properties to those used in the experiments performed by Krochak et al. (2008). These suspensions contained fibers of length $L = 5$ mm, diameter, $d = 0.1$ mm and of concentration $nL^3 = 8$. The fiber aspect ratio, r, that is, the ratio of fiber length, L to its diameter, d, is 50. The Reynolds number, based on the length of the fiber is asymptotically small and based on the inlet channel height is approximately 500.

Controlling the orientation state of fiber suspensions in tapered channel flows is of major interest to papermaking. During papermaking, a semi-dilute fiber suspension flows through a specially shaped duct called a headbox. The first section of the headbox consists of a manifold that sets up a uniform flow across the duct. The flocculated fiber suspension is then fluidized by turbulence created locally from a sudden change in geometry just after the manifold. This is indicated in Fig. 1 as the turbulence generators. The fluidized fiber suspension subsequently passes through a planar contraction called the nozzle, which accelerates the fluid to a high speed and creates a thin planar jet. The jet is typically 10 m wide, 1 cm thick with a mean velocity in excess of 20 m/s. The jet then impinges on a permeable mesh where the water is drained and

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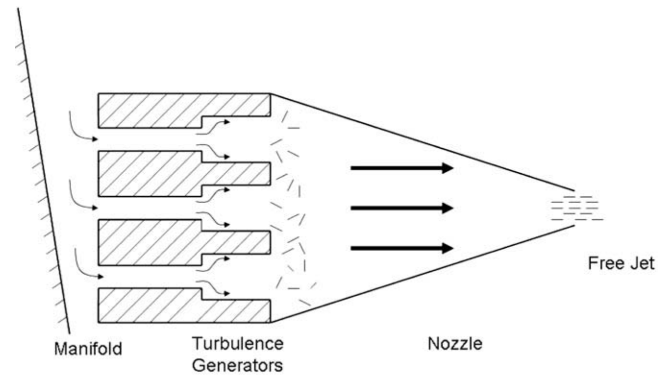

Fig. 1. A generalized industrial headbox.

the paper sheet formed. The orientation distribution of the pulp fibers on the forming mesh plays a fundamental role in determining the strength of the final product.

Fiber orientation in paper depends on a number of different factors, such as the fiber orientation state at the contraction inlet, the concentration of fibers in the suspension, and perhaps most importantly, on the flow field generated after the turbulence generator. Major theoretical developments in fiber suspension rheology have been made over the last two decades. Perhaps most notably, it has been established that the suspension rheology and flow field respond to the orientation state of the suspension (e.g. Batchelor, 1970; Cox, 1970; VerWeyst and Tucker, 2002; Lipscomb and Denn, 1988). The result is a two-way coupling between the fiber orientation state and the underlying flow field. The first to address this issue was Batchelor (1970) who developed a general constitutive equation for the bulk stress in a suspension of rigid, inertialess particles of arbitrary shape in a Newtonian fluid. By representing a single particle in suspension as a distribution of Stokeslets over a line enclosed by the particle body, Batchelor determined expressions for the resultant force required sustaining translational motion and the resultant couple required to sustain rotational motion. Dinh and Armstrong (1984) extended Batchelor's theory to account for the orientation state of elongated particles and its effect on the bulk stress within the suspension. This was accomplished by assuming that the orientation state of the suspension can be completely described by a known orientation distribution function, Ψ, such that the probability of finding fibers oriented between the angles ϕ and $\phi + \partial\phi$ is $\Psi(\phi)\partial\phi$. By linearizing the flow field around the particle they were able to equate Batchelor's constitutive equation to a new constitutive equation; one that is proportional to the fourth-order moment tensor of Ψ. The proportionality constant is referred to as the effective viscosity of the suspension. Shaqfeh and Fredrickson (1990) derived asymptotic expressions for the effective viscosity of dilute and semi-dilute suspensions of rods in a Newtonian fluid. For semi-dilute fiber suspensions, they express the fiber stress as follows:

$$\tau^{\text{fiber}} = \frac{\mu c r^2 \dot{\gamma} : \langle \mathbf{pppp} \rangle}{\ln(1/c) + \ln(\ln(1/c)) + 1.439} \tag{2}$$

where c is the volume fraction of fibers within the suspension which can be related to the concentration parameter, nL^3 as $c = \frac{4\pi n L^3}{3r^2}$; μ is the viscosity of the suspending fluid and $\dot{\gamma}$ is the fluid strain rate tensor, defined as

$$\dot{\gamma} = (\nabla \mathbf{u} + \nabla \mathbf{u}^{\mathbf{T}}) \tag{3}$$

The remaining term that needs to be defined in Eq. (2) is the fourth-order moment of the orientation distribution function Ψ. It is often referred to as the fourth-order orientation tensor and is defined as

$$\langle \mathbf{pppp} \rangle = \int p_i p_j p_k p_l \Psi(\phi)\, d\phi \tag{4}$$

where $\mathbf{p}$ is a unit vector pointing in the direction parallel to the axis of the fiber, that is

$$\mathbf{p} = \begin{bmatrix} \cos\phi \sin\theta \\ \sin\phi \sin\theta \\ \cos\theta \end{bmatrix} \tag{5}$$

where ϕ is the projected angle of the fiber in the xy-plane and θ is the angle between the fiber and the z-axis, see Fig. 2.

The analytic theory of fiber motion in Newtonian flows is also well established. Jeffery's equation of motion (Jeffery, 1922) for a single rigid ellipsoid in an unbounded flow forms the basis for most of this work. For cases above the dilute limit, quantitative relationships between the suspension orientation state and processing conditions have shown that the problem formulation should account for the fact that fibers orient in response to gradients in the flow and disorient in response to hydrodynamic fiber–fiber interactions (e.g. Koch, 1995; Folgar and Tucker, 1984; Rahnama et al., 1995; Altan et al., 1989; Lipscomb and Denn, 1988; Jackson et al., 1985). To help address this issue, Folgar and Tucker (1984) model fiber–fiber interactions as randomly occurring events resulting in a behavior which seemingly mimics a diffusion-type process. In this approach, these authors use an empirically determined rotary diffusion coefficient, D_r, whose value is unknown *a priori* and must be determined through experiment. They proposed, through dimensional analysis, a simple relationship in which D_r is linearly proportional to the magnitude of the rate of strain tensor, $\|\dot{\gamma}\|$. For two dimensional flow in a linear contraction, D_r can be expressed as

$$D_r = C_I \|\dot{\gamma}\| \tag{6}$$

where C_I is traditionally called the interaction coefficient and is related to a number of suspension parameters such as concentration, aspect ratio, and fiber length.

Recently, researchers have been making great efforts to perform 3D fiber orientation predictions for 2D and 3D flows inside complex geometries using a fully coupled model of fiber orientation (e.g. VerWeyst and Tucker, 1999, 2002; Lipscomb and Denn, 1988; Lin and Zhang, 2002). The difficulty with this approach lies in the large computational domain required to resolve both the spatial and orientation domains when directly computing the orientation distribution function. In order to deal with this problem, researchers have had to rely on the use of orientation tensors to predict fiber orientation as opposed to a direct computation of the orientation distribution function (e.g. Jackson et al., 1985; VerWeyst and Tucker, 1999, 2002). The second-order orientation tensor, $\langle \mathbf{pp} \rangle$, for the orientation distribution function, Ψ, contains

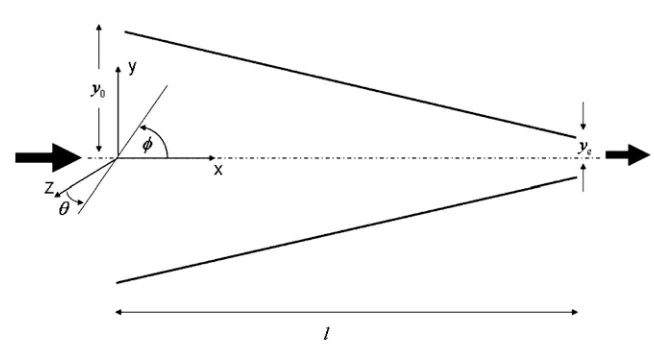

Fig. 2. The orientation of a fiber with respect to flow in a linear contraction. ϕ is the angle of the fiber projected into the xy-plane and the θ is the angle of the fiber with respect to the z-axis.