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Short communication

Cohesion and tensile strength estimation from incomplete shear analysis data

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ABSTRACT

The Yield Loci Curves (YLC's) of compacted cohesive powder specimens, from which its Flow Function (FF) is derived, are frequently described by the Warren Spring equation whose parameters are the cohesion (*C*), tensile strength (*T*) and curvature index (*n*). Ideally, for each pre-consolidation level, *T* should be determined independently with a special instrument, and *C* and *n* extracted from the experimental yield loci measurements by non-linear regression using the Warren Spring equation as a model. There are situations, however, where direct determination of *T* is not a feasible option, and the number of experimental yield stress data, limited by logistic considerations, can be insufficient for meaningful regression. In such cases, *T*, *C* and *n* can be estimated with a freely downloadable interactive Wolfram Demonstration, with which three (the theoretical minimum) or more experimental yield loci are matched with a generated YLC using the Warren Spring equation as a model. To obtain a match, one moves *T*, *C* and *n* sliders on the screen using the calculated Mean Squared Error (MSE) as a guide and for fine-tuning. At least in principle, once *T*, *C* and *n* have been estimated in this way, they can be used to calculate the corresponding principal stresses and effective internal friction angle. The method is demonstrated with published experimental shear data on wetted glass beads and limestone powder.

1. Introduction

Cohesive powders' flowability is commonly determined by shear analysis of compacted specimens. The data, in the form of a set of Yield Loci Curves (YLC's), are used to determine the major consolidation and corresponding unconfined yield strength, which in turn can be used to calculate the powder's Flow Function [1–4]. The major consolidation and unconfined yield stresses, σ_1 and σ_c, are extracted from the Mohr semi-circles that are tangent to the YLC and pass through the initial consolidation point (σ_0, τ_0) and the plot's origin (0,0), respectively. Construction of the Mohr semi-circles can be done manually or calculated numerically whenever the YLC can be expressed algebraically [5]. To determine and plot a complete YLC requires shearing data obtained under low normal stresses in order to determine the compact's cohesion *C* by extrapolation, or by interpolation if the compact's tensile strength *T* has been determined independently with special instrumentation. For technical and logistic considerations, fulfillment of these requirements is not always feasible in routine testing, which begs the question of whether *C*, *T* and *n* can be estimated from shear data obtained under relatively high stresses, i.e., to estimate *T* without direct experimental determination and *C* without interpolation. The objective of this communication is to suggest a mathematical method to obtain such estimates and demonstrate its performance with a freely downloadable user-friendly interactive Wolfram Demonstration especially written for the purpose.

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2. The method

The YLC of many powder compacts pre-consolidated under the same normal stress has been described by the Warren Spring equation [6]:

$$\left(\frac{\tau}{C}\right)^n = \frac{\sigma + T}{T} \quad (1)$$

or in the explicit $\tau(\sigma)$ vs. σ form

$$\tau(\sigma) = C\left(\frac{\sigma + T}{T}\right)^{\frac{1}{n}} \quad (1a)$$

where $\tau(\sigma)$ is the shear stress, σ the normal stress, *C* the cohesion and *T* the tensile strength, all having the same stress units, e.g., kPa, and *n* a dimensionless curvature index.

In principle, the Warren Spring equation's three parameters can be extracted from three experimental points along the YLC, (σ_1,τ_1), (σ_2,τ_2) and (σ_3,τ_3), by solving the three simultaneous algebraic equations $\tau_1 = \tau(\sigma_1)$, $\tau_2 = \tau(\sigma_2)$ and $\tau_3 = \tau(\sigma_3)$, where $\tau = \tau(\sigma)$ is described by Eq. (1a) and *C*, *T* and *n* are the three unknowns. Had there been neither experimental data scatter nor the slightest deviation from the mathematical model, experimental determination of the entire

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YLC would have become unnecessary. This, of course, is rarely if ever the case and hence more experimental data points are needed. In principle, one could extract C, T and n from a set of more than three experimental data points, which does not include the points $(0, C)$ and $(-T, 0)$ by nonlinear regression, using Eq. (1a) as a model. But here too, the regression might not always render realistic parameter values from a few experimental data points because of their scatter and/or potential existence of slight deviations from the assumed Warren Spring model [7,8]. Moreover, both the numerical solution and nonlinear regression require close initial guesses of the sought parameters' values for the method to work and render realistic parameter values. Coming up with close initial guesses might not be always easy, and it is not guaranteed that using them will always render a unique and/or meaningful solution.

A way to circumvent these difficulties is to plot the three or more $\tau(\sigma)$ vs. σ experimental data points and match them visually with a YLC generated with the Warren Spring equation through adjustment of the numerical values of its C, T and n parameters. This can be done with the Manipulate function of Mathematica® (Wolfram Research, Champaign, IL, USA), the program used in this work. An interactive program to do the matching, which includes instructions of how to use it and selective examples, has been posted on the Internet, and is available as a freely downloadable Wolfram Demonstration – open: http://demonstrations.wolfram.com/EstimatingCohesionAndTensileStrengthOfCompactedPowders/. [The Wolfram CDF Player, the program that runs the Demonstration and over 11,450 other Wolfram Demonstrations to date, is also freeware and can be downloaded from the Internet following instructions on the screen. Any Wolfram Demonstration's code can be freely downloaded too, but it can only be modified with Mathematica®, the language in which it has been written.]

Fig. 1 shows the Wolfram Demonstration's screen display when opened. It depicts a simulated set of six experimental (σ_i, τ_i) points, matched with YLC generated with the Warren Spring equation as a model. The corresponding C, T and n parameters' magnitudes, indicated by their respective sliders' positions, in this case the default values, are also displayed on top of the plot together with the calculated Mean Squared Error (MSE). The MSE serves as a comparative statistical measure of the match's closeness, and hence can be used as a guide during the search for a match, and in the parameters' fine-tuning once visual match has been obtained.

3. Matching experimental data with the Warren Spring equation

Figs. 2 and 3 show published experimental shear tests data on wetted glass beads having two moisture contents [9] and two size fractions of a BCR limestone powder [10]. Superimposed on the data points are the YLC's obtained by the matching method. The two figures also show the rounded C, T and n estimates extracted from their slider positions and accompanying small MSE values. The corresponding compacts' unconfined yield strengths (σ_c's) of the glass beads were 1.2 & 1.7 kPa, and of the limestone powders 2.2 & 2.7 kPa. These were calculated with the interactive Wolfram Demonstration entitled "Principal Stresses in Compacted Cohesive Powders" – open: http://demonstrations.wolfram.com/PrincipalStressesInCompactedCohesivePowders/ using the estimated C, T and n values as shown in Fig. 4. The two figures (Figs. 2 and 3)

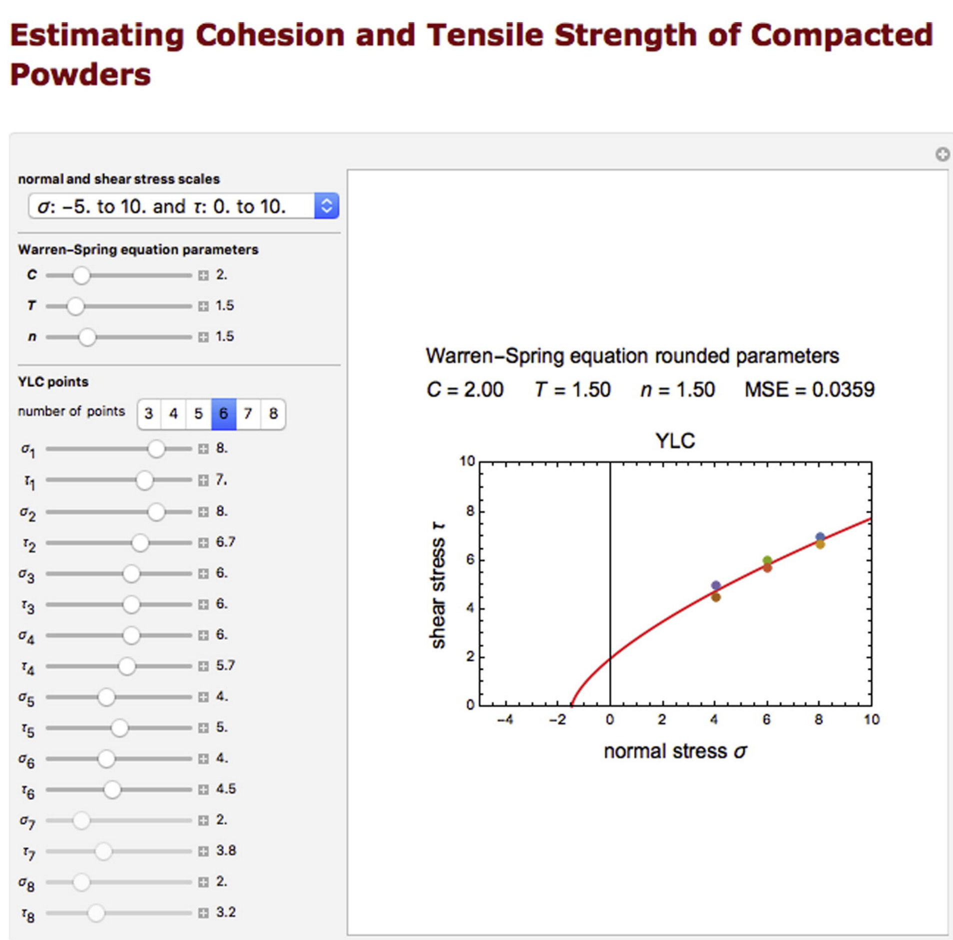

Fig. 1. Screen display of the freely downloadable Wolfram Demonstration that estimates the Warren Spring equation's parameters C, T and n from experimental yield shear data with the matching method.