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Boundary layer flow analysis of a nanofluid past a porous moving semi-infinite flat plate by optimal collocation method

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ABSTRACT

A novel concept of collocation method is introduced and used for the first time to obtain a simple and accurate solution for the boundary layer in unbounded domain. Similar analysis is however not available yet in the literature. The idea is to transform the equations and boundary conditions into another set of variables and also to augment an extra boundary condition. To demonstrate the effectiveness of this ideal, we apply optimal collocation method (OCM) to find the approximate solution for the boundary layer flow of a nanofluid past a porous moving semi-infinite flat plate. The influence of pertinent parameters on the flow field characteristics is studied. The obtained results have been compared with the numerical solutions from fourth order Runge–Kutta method. The solution shows that the results of the present method are in excellent agreement with those of the numerical one. It is important that we applied OCM for the problem in unbounded domain without using Pade approximants, perturbation, linearization, small parameter or auxiliary parameter. The approach used in the present work can also be extended to different nonlinear problems in unbounded domain and the conclusion is considered useful for engineering applications involving infinity domain.

1. Introduction

In various fields of science, there are few phenomena occurring linearly. In most cases, scientific problems are inherently of nonlinearity. Nonlinear problems [1–3] play important roles in fluid mechanics and heat transfer. With the rapid development of nonlinear science, many different methods were proposed to solve various boundary value problems (BVP), such as the Adomian's Decomposition Method (ADM) [4–8], the Variational Iteration Method (VIM) [9–12], the Homotopy Perturbation Method (HPM) [13–16] and Homotopy Analysis Method (HAM) [17–20]. Many problems in science and engineering arise in unbounded domains. Due to the existence of infinite boundary values, nonlinear differential equations which arise in boundary layer problem are often very complicated and there are some restrictions for them to be solved analytically. For example, there is a special value for η_∞ which changes with the change in flow parameters. The above mentioned methods fail to estimate the numerical value of infinity for problems such as boundary layer problem which has boundary condition at infinity. Considering the shortcomings mentioned above, researchers have been looking for alternative ways to solve these problems. Optimal collocation method, a special technique that can transform the equations and boundary conditions and augment an extra boundary condition, have been proposed.

Collocation (CM), Least Square (LSM) and Galerkin (GM) methods which are examples of the Weighted Residuals Methods (WRMs) were firstly introduced by Ozisik [21] for solving differential equations in heat transfer problems. Stern and Rasmussen [22] used CM procedure for solving a third order linear differential equation. Ganji et al. [23] solved the problem of the laminar nanofluid flow in a semi-porous channel in the presence of transverse magnetic field by LSM and GM. Recently Ganji et al. [24–25] used CM for heat transfer study through porous fins. Also Ganji and Hatami analyzed least-squares approximations for the heat transfer and nanofluid flow between two coaxial cylinders [26], two vertical flat plates [27] and parallel disks [28]. Although GM, LSM and CM have been applied by Ganji for solving nonlinear fluid mechanics and heat transfer equations in the finite domain, such analysis has not been used for the infinite domain $0<\eta<\infty$. The aim of this study is to apply developed analytic method, the optimal collocation method (OCM), to eliminate the infinite boundary values. For this purpose, after brief introduction of OCM, we apply OCM to find the approximate solution for the problem of the boundary layer flow of a nanofluid [29–30] past a porous moving flat plate. Fluids added with nano-scale particle are called as nanofluid, which was first proposed by Choi [31]. They found that the addition of a small amount (less than 1% by volume)

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Table 1
Thermo physical properties of water and nanoparticles.

	Pure water	Copper (Cu)
$\rho(\mathrm{kg}/m^3)$	997.1	8933
$k(\mathrm{w}/mK)$	0.613	400
$\alpha \times 10^{-7}(\mathrm{m}^2/\mathrm{s})$	1.47	1163.1

of nanoparticles to conventional heat transfer liquids increased the thermal conductivity of the fluid up to approximately two times. The nanofluids have many applications in the industries such as heat exchangers, lubricants, micro-electromechanical systems, and micro-channel heat sinks. In the current study, the velocity profiles are obtained and the effects of uniform suction/injection parameter, the velocity ratio parameter and nanoparticle volume fraction on the flow field characteristics are analytically studied by using OCM and are shown graphically through a set of graphs. The obtained results are also compared with numerical results which are solved by Maple software using fourth order Runge–Kutta method. The skin friction stress on the wall has also been studied. The main aim of our present study is to highlight the optimal collocation method in analysis of boundary layer flow of a nanofluid past a porous moving semi-infinite flat plate. It is a quite complex case and we thus use a single phase model for simplicity. Our study shows the capability and effectiveness of OCM and exhibits application of this method to eliminate the infinite boundary values for solution of boundary layer problems.

2. Governing equations

We consider a two-dimensional steady laminar boundary layer flow over a porous moving semi-infinite flat plate in a water based nanofluid containing Cu (copper). It is assumed that the base fluid and the nanoparticles are in thermal equilibrium and no slip occurs between them. The thermo-physical properties of Cu nanoparticles and base fluid are shown in Table 1 [32]. The plate is assumed to move with constant velocity U_w in the same direction to the free stream U_∞. The physical sketches are given in Figs. 1 and 2. Using boundary layer approximation, under zero pressure gradient, the equations for such type of flow equation may be written in usual notation as:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{1}$$

$$\rho_{nf}\left[u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}\right]=\mu_{nf}\frac{\partial^2 u}{\partial y^2}. \tag{2}$$

The boundary conditions for the above equations are:

$$y=0 \ \Rightarrow u=U_w, v=V_w \tag{3}$$

$$y\rightarrow\infty\Rightarrow u\rightarrow U_\infty \tag{4}$$

where u and v are the velocity components in x and y directions respectively, μ_{nf} is the viscosity of the nanofluid and ρ_{nf} is the density of the nanofluid, which are given [32]:

$$\mu_{nf}=\frac{\mu_f}{(1-\varphi)^{2.5}} \tag{5}$$

$$\rho_{nf}=(1-\varphi)+\varphi.\rho_s \tag{6}$$

where ρ_f is the density of fluid, ρ_s is the density of nanoparticles and φ is defined as the volume fraction of the nanoparticles. For simplicity of basic equations of considered problem, bellow transformations are used:

$$\eta=\frac{y}{\sqrt{\frac{\upsilon_f x}{(U_w+U_\infty)}}} \tag{7}$$

$$f=\frac{-\psi(x,y)}{\sqrt{(U_w+U_\infty)\upsilon_f x}} \tag{8}$$

where the stream function ψ is defined with $u=-\frac{\partial\psi}{\partial y}$ and $v=\frac{\partial\psi}{\partial x}$ and f is the similarity functions dependent on η. In order to ensure the existence of similarity solutions of Eqs. (1) and (2), we take $V_w=-\frac{f(0)}{2}\sqrt{\frac{(U_w+U_\infty)\upsilon_f}{x}}$. Applying the above transformations leads to the reduction of basic equations as below:

$$f'''(\eta)+\frac{1}{2}\left((1-\varphi)^{2.5}\left[(1-\varphi)+\varphi.\frac{\rho_s}{\rho_f}\right]\right)f(\eta)f''(\eta)=0. \tag{9}$$

It subjects to the boundary conditions:

$$\eta=0 \ \Rightarrow \ f=s \tag{10}$$

$$\eta=0 \ \Rightarrow \ f'=\varepsilon \tag{11}$$

$$\eta\rightarrow\infty \ \Rightarrow f'=1-\varepsilon \tag{12}$$

where $f(0)=s$ is the suction/injection parameter that is a non-dimensional quantity, with $s<0$ for injection, $s>0$ for suction and $s=0$ for

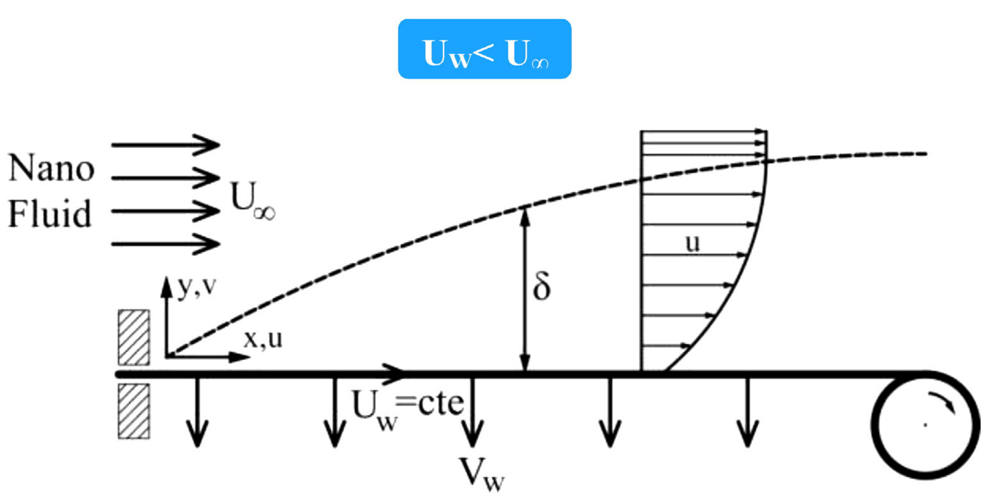

Fig. 1. Geometry of problem when $0<\varepsilon<0.5(U_w<U_\infty)$.