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ScienceDirect
International Journal of Multiphase Flow 34 (2008) 61–75
International Journal of Multiphase Flow
www.elsevier.com/locate/ijmulflow

Purely irrotational theories for the viscous effects on the oscillations of drops and bubbles

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Received 11 June 2006; received in revised form 16 May 2007

Abstract

In this paper, we apply two purely irrotational theories of the motion of a viscous fluid, namely, viscous potential flow (VPF) and the dissipation method to the problem of the decay of waves on the surface of a sphere. We treat the problem of the decay of small disturbances on a viscous drop surrounded by gas of negligible density and viscosity and a bubble immersed in a viscous liquid. The instantaneous velocity field in the viscous liquid is assumed to be irrotational. In VPF, viscosity enters the problem through the viscous normal stress at the free surface. In the dissipation method, viscosity appears in the dissipation integral included in the mechanical energy equation. Comparisons of the eigenvalues from VPF and the dissipation approximation with those from the exact solution of the linearized governing equations are presented. The results show that the viscous irrotational theories exhibit most of the features of the wave dynamics described by the exact solution. In particular, VPF and DM give rise to a viscous correction for the frequency that determines the crossover from oscillatory to monotonically decaying waves. Good to reasonable quantitative agreement with the exact solution is also shown for certain ranges of modes and dimensionless viscosity: For large viscosity and short waves, VPF is a very good approximation to the exact solution. For 'small' viscosity and long waves, the dissipation method furnishes the best approximation.

Keywords: Drops; Bubbles; Potential flow; Two-phase flow; Gas–liquid flow; Viscous potential flow

1. Introduction

A viscous liquid drop surrounded by a quiescent gas or a gas bubble immersed in a viscous liquid tends to an equilibrium spherical shape if the effects of surface tension are significantly large in comparison with gravitational effects. When the spherical interface of the bubble or drop is slightly perturbed by an external agent, the bubble or drop will recover their original spherical configuration through an oscillatory motion of decreasing amplitude. In the case of the drop, depending upon its size and physical properties, the return to the

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0301-9322/$ - see front matter
doi:10.1016/j.ijmultiphaseflow.2007.06.008

spherical shape may consist of overdamped aperiodic waves that decrease monotonically. For a drop immersed in another viscous liquid, decaying oscillatory waves always occurs at the liquid–liquid interface.

Early studies on the subject for inviscid liquids are due to Kelvin (1890) and Rayleigh (1896). Lamb (1881) considered fully-viscous effects on the oscillations of a liquid spheroid by solving the linearized Navier–Stokes equations. Applying Stokes' ideas (Stokes, 1851), Lamb also approximated the effect of viscosity on the decay rate of the oscillations on a liquid globule by means of the dissipation method, in which an irrotational velocity field is assumed. His result is independent of the nature of the forces that drive the interface to the spherical shape. Furthermore, Lamb used energy arguments to compute the frequency of the oscillations governed by self-gravitation in the absence of viscosity, recovering the result due to Kelvin. In his book on hydrodynamics, Lamb (1932) included his dissipation calculation of the decay rate of the oscillations on a spherical globule and added the corresponding result for a spherical bubble in a viscous liquid. Chandrasekhar (1959) studied fully-viscous effects on the small oscillations of a liquid globe with self-gravitation forces neglecting surface tension. The same form of the solution was also obtained by Reid (1960) when surface tension instead of self-gravitation is the force that tends to maintain the spherical shape. A good account of both solutions is presented in the treatise by Chandrasekhar (1961). Following Lamb's reasoning, Valentine et al. (1965) applied the dissipation method to the case of a drop surrounded by another viscous liquid. They presented their result for two fluids with the same density. However, the dissipation approximation for the two-liquid case, according to Miller and Scriven (1968), underestimates the decay rate.

Comprehensive analyses of viscous effects in a drop embedded in liquid were presented by Miller and Scriven (1968) and Prosperetti (1980a) using normal modes. The latter found a continuous spectrum of eigenvalues for an unbounded outer fluid. Prosperetti (1977, 1980b) considered the initial-value, fully-viscous problem posed by small perturbations about the spherical shape of a drop or a bubble with no assumption about the form of the time dependence. The solution showed that the normal-mode results are recovered for large times.

Finite size disturbances have received some attention. Tsamopoulos and Brown (1983) considered the small-to-moderate-amplitude inviscid oscillations using perturbation methods. Lundgren and Mansour (1988) and Patzek et al. (1991) studied the inviscid problem posed by large oscillations applying the boundary-integral and the finite-element methods, respectively. Lundgren and Mansour also investigated the effect of a 'small' viscosity on drop oscillations. Basaran (1992) carried out the numerical analysis of moderate-to-large-amplitude axisymmetric oscillations of a viscous liquid drop.

In this paper, approximate solutions of the linearized problem for small departures about the spherical shape for a drop surrounded by a gas of negligible density and viscosity or a bubble embedded in a liquid are sought using viscous potential flow (VPF) and the dissipation method. VPF is a purely-irrotational-flow theory in which viscosity enters the problem through the viscous normal stress at the interface (Joseph and Liao, 1994a,b; Joseph, 2003). If the viscosities of the fluids are neglected, the analysis reduces to inviscid potential flow (IPF). Another viscous irrotational approximation can be obtained by applying the dissipation method (Joseph and Wang, 2004). In this approximation, which requires the evaluation of the mechanical energy equation, viscous effects are accounted for through the computation of the viscous dissipation originated by the irrotational flow. In addition to his study of a viscous globule and a gas bubble, the dissipation method was used by Lamb (1932) in his analysis of the effect of viscosity on the decay of free gravity waves. He found the decay rate from the dissipation method in complete agreement with the exact solution for the decay rate of free gravity waves for 'small' viscosity. However, his analysis did not render viscous effects for the frequency of the waves. The dissipation calculation presented here does give rise to a viscous correction for the wave frequency, thus predicting a crossover from oscillatory to monotonically decaying waves.

VPF was used by Funada and Joseph (2002) to study the problem of capillary instability. Their results for the growth rate were in much better agreement with Tomotika's (1935) exact normal-mode solution than inviscid potential flow. Wang et al. (2005a) computed the growth rates for this configuration by adding a viscous correction to VPF, when either the interior or exterior fluid is a gas of negligible density and viscosity. They found good agreement between their results and the exact solution. The case of capillary instability of two viscous liquids was considered by Wang et al. (2005b), obtaining good to reasonable agreement for the maximum growth rates whereas poor agreement for long waves. Wang et al. (2005a,b) also used the dissipation method in the problem of capillary instability and obtained the same growth rate as the viscous correction