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Brief communication

Modelling the wetting of a solid occlusion by a liquid film

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ABSTRACT

We investigate in this work how the presence of an occlusion affects the dynamics of the wetting front of a liquid film draining down a vertical surface. This numerical study is developed in the context of the lubrication approximation. Through a parametric study, we show that depending on the asymptotic film thickness and the fluid properties, there exists a critical substrate contact angle below which separation of the contact line from the occlusion wall is observed which results in the appearance of a dry zone in the wake of the occlusion. In analogy with external aerodynamics, we also show that a sharp corner in the occlusion can induce this contact line separation. Our numerical results also highlight the importance of the occlusion wettability on the morphology of the wetting front suggesting a possible mechanism to control and mitigate the often undesirable fingering instability.

1. Introduction

The deposition of a thin liquid layer on a solid substrate is an important part of many manufacturing processes. Beyond classical coating processes, applications can be found in industries as diverse as microelectronics, displays, optical storage, or microfluidic devices. Typically, the fluid is driven on the substrate by an applied surface stress or a body force such as gravity and the aim is to obtain a defect-free coated layer. In practice, achieving this goal on a smooth, flat substrate already proves to be a challenging task because of the inherent instability of the contact line and the inevitable presence of surface contaminations. These contaminations may be of a chemical or topographical nature. It is known, for example, that holes in a horizontal sheet of fluid on a partially wetting substrate spontaneously expand if their radius is greater than a threshold value and close up otherwise, (Taylor and Michael, 1973; Sharma and Ruckenstein, 1989). An other undesirable source of defect is the fingering instability which leads to the formation of narrow rivulet of liquid that penetrates into the unwetted part of the substrate. This instability has been extensively studied and a formal linear stability analysis performed (Troian et al., 1989; Spaid and Homsy, 1996; Bertozzi and Brenner, 1997). The presence of a hydrophobic patch on the substrate is also known to produce a dry arch downstream, (Podgorski et al., 1999; Ye and Chang, 1999; Marshall and Wang, 2005), with highly undesirable consequences in heat exchangers design, for example. Overall the research efforts have been mainly focused on film rupture mechanisms related to wetting phenomena.

In contrast, the interaction of a contact line with an occlusion which fully protrude from the liquid layer has received little attention. It is however highly relevant to coating processes and provided the impetus for the research presented here. This problem is the small scale equivalent of a flood front impacting on a bridge pillar (Chaplin and Teigen, 2003; Shao et al., 2013) and the coating analogue would be that of a layer of paint flowing past a nail. Another important application is that of lubricating films in engines which inevitably encounter such occlusions. In spite of its clear important applications, this problem is yet to be investigated in a systematic way. This study is a first attempt to model such flows with a particular emphasis on the conditions which promote film rupture.

This problem lays at the cross-road between two distinct research areas which have received much attention.

- **Interaction of a contact line with a patch of poor wettability.** This problem has been explored experimentally, theoretically and numerically and has a strong motivation stemming from the design of heat exchangers, El-Genk and Saber (2001). Indeed the dry region developing downstream of the hydrophobic patch has an important detrimental effect to the overall heat transfer. It has been studied experimentally by Podgorski et al. (1999), Rio and Limat (2006), and Sebilleau et al. (2009) and theoretically, a fine example being the papers of Wilson and co-workers describing the shape of a slender dry patch in a liquid film draining under gravity down an inclined plane, (Wilson et al., 2001; Yatim et al., 2013). Much work related to the motion of a contact line on a substrate with variable wettability was performed numerically in the context of the

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lubrication approximation which reduces the three-dimensional problem fluid dynamics problem to a two-dimensional one by averaging quantities across the thickness of the fluid layer. Important contributions include for example Kondic and Diez (2004), Schwartz (1998), Schwartz and Eley (1998), Marshall and Wang (2005), Zhao and Marshall (2006), Gaskell et al. (2004) to name but a few.

- **Fully-developed thin liquid film past solid occlusion.** This problem has been the topic of recent numerical studies which address the problem of computing the free surface profile of a film surrounding a protruding occlusion. To the best of the author's knowledge, the earliest reported modelling attempt is the work of the present author, Sellier (2006), which involves a formulation based on the lubrication approximation which is solved using Finite Elements in the commercial multiphysics package COMSOL. This study was then followed by a series of papers based on the same formulation exploring the numerical solution technique, Lee et al. (2008), the addition of more complex physics, Lee et al. (2011), and the effect of inertia, Veremeiev et al. (2011). At the same time, Baxter and co-workers investigated the same problem but the approach they adopted is based on solving the Stokes equations using a Boundary Element technique, Baxter et al. (2009, 2010). Naturally, this latter method has less restrictive assumptions than the lubrication approximation.

This paper focuses on how the contact line interacts with a simple occlusion with a strong emphasis on conditions likely to lead to film rupture. This problem is investigated in the lubrication approximation framework. The next section specifies the problem, states the governing equations, and highlights the numerical solution procedure. The following section reports the results of this investigation and the final section builds on a discussion of the results to draw concluding remarks.

2. Problem specification and solution

We consider here the flow of a thin liquid film draining down a vertical surface as illustrated in Fig. 1. In the following, upper and lower case notations stand for dimensional and dimensionless quantities, respectively. The fluid has viscosity μ, density ρ, and surface tension σ. The coordinates (X, Y) are in the substrate plane, X being downslope in the flow direction. The flow is described in the lubrication approximation framework which assumes negligible inertia and small free surface slope. Assuming that the Reynolds number is of $\mathcal{O}(1)$, at most, then the inertial terms enter the governing equation at $\mathcal{O}(\epsilon)$ where $\epsilon \ll 1$ and can be neglected, see Craster and Matar (2009). This approximation which has been used extensively to analyze a range of thin film flow problems provides a pair of non-linear evolution equations for the film thickness $H(X, Y, T)$ and the pressure $P(X, Y, T)$ at the film free surface. A complete derivation of the lubrication approximation is beyond the scope of this paper, only the main equations are reported here. The interested reader is referred to Leal (2010) for more details on the derivation of these equations. Conservation of mass requires that

$$\frac{\partial H}{\partial T} = -\nabla \cdot \vec{Q}, \tag{1}$$

where $\vec{Q}$ is the discharge vector which is obtained by integrating the velocity vector $\vec{V}$, assumed to be parabolic to leading order, across the film as follows

$$\vec{Q} = \int_0^H \vec{V}\, dZ = -\left(\frac{H^3}{3\mu}\left(\nabla P - \rho g\, \vec{i}\right)\right), \tag{2}$$

where $\vec{i}$ is the unit vector pointing in the x-direction. The pressure at the free surface is obtained by considering the normal stress balance condition at the interface. To leading order, the film free surface pressure is made up of two main contributions: the capillary pressure and the disjoining pressure Π. The latter is introduced as a possible mean to alleviate the singularity which arises at the contact line. Accordingly,

$$P = -\sigma\nabla^2 H - \Pi(H, H^\star), \tag{3}$$

The disjoining pressure model which was introduced by Schwartz and Eley (1998) assumes a thin precursor film of thickness $H^\star$ ahead of the wetting line and relates the observed contact angle for partially wetting systems to intermolecular forces that become important for liquid layers of submicroscopic dimensions. The disjoining pressure term is given by

$$\Pi(H, H^\star) = B\left[\left(\frac{H^\star}{H}\right)^n - \left(\frac{H^\star}{H}\right)^m\right], \tag{4}$$

where n and m are the exponents of the interaction potential and B is a constant. Here, we use for B the expression derived in Zhao and

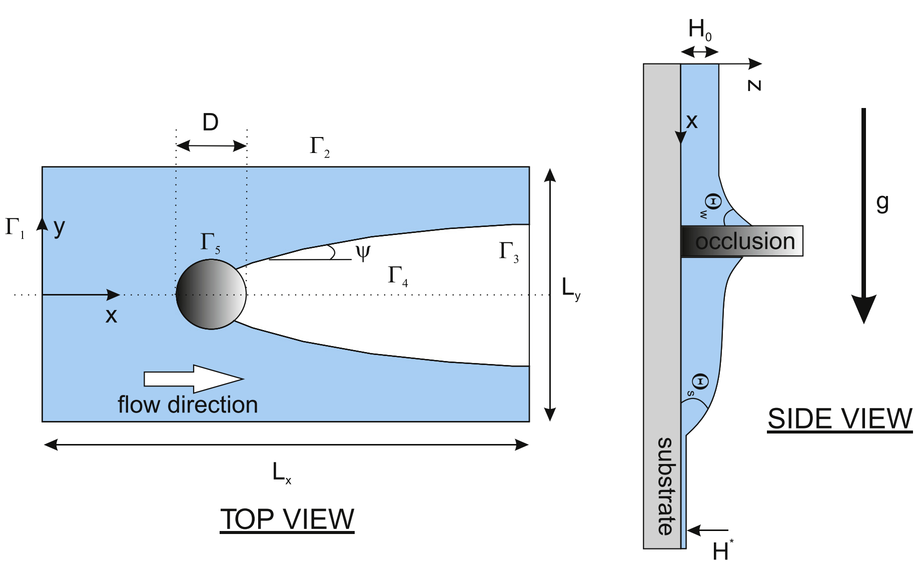

Fig. 1. Computational domain and notations.