International Journal of Multiphase Flow 71 (2015) 94–97

Contents lists available at ScienceDirect

International Journal of Multiphase Flow

journal homepage: www.elsevier.com/locate/ijmulflow

Brief Communication

A comparison for different wall-boundary conditions for kinetic theory based two-fluid models

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ARTICLE INFO

Article history:
Received 27 September 2014
Received in revised form 24 November 2014
Accepted 29 December 2014
Available online 19 January 2015

Keywords:
Gas–particle flow
Boundary conditions
Particle–wall collision
Eulerian model

Introduction

This research-note discusses the different boundary conditions for the solids wall shear stresses and the corresponding flux of fluctuation energy (Johnson and Jackson, 1987; Jenkins and Louge, 1997; Li and Benyahia, 2012; Schneiderbauer et al., 2012a), which are most commonly employed in kinetic theory based two-fluid simulations of wall bounded gas–solid flows. This study further evaluates the applicability of those models to different gas–solid flow regimes from the no-sliding to the all-sliding limit. The comparison reveals that the boundary conditions of Li and Benyahia (2012) and Schneiderbauer et al. (2012a) yield the best predictions of the solids wall shear stresses when plotted against the discrete element results of Louge (1992); in the case of the flux of fluctuation energy the work of Schneiderbauer et al. (2012a) appears to be the most developed theory with respect to the reference data of Louge.

In many industrial processes, wall bounded gas–solid flows are of crucial importance. On the one hand the powder may be conveyed from a feeding vessel towards its processing point (Schellander et al., 2013); on the other hand solid particles may be processed in fluidized beds, risers as well as moving beds (Schneiderbauer et al., 2012b and references cited therein). In these processes, the movement of the solid particles is strongly affected by the collisions of the particles with the confining walls, which motivated many researchers to study and analyze the details of the particles behavior under the effect of wall-roughness and wall-friction. In the case of kinetic theory based two-fluid models (TFM) particle–wall collisions are characterized by the transfer of momentum and pseudo thermal energy. In the past decades different authors attacked this problem and proposed various theories to determine the impact of the wall, which may be characterized by wall friction and wall roughness, on the gas–solid flow in TFM (Johnson and Jackson, 1987; Jenkins, 1992; Jenkins and Louge, 1997; Li and Benyahia, 2012; Schneiderbauer et al., 2012a).

The most commonly used theory for these solids boundary conditions, these are the solids wall shear stresses and flux of fluctuation energy, was presented by Johnson and Jackson (1987). Here, the effect of momentum transfer and large-scale wall-roughness is characterized by a single factor referred to as specularity coefficient. Furthermore, the flux of fluctuation energy was determined from the balance between the work done due to particle–wall collisions and the dissipation due to the inelasticity of the particles (compare with Table 1, Eqs. (1) and (2)).

Jenkins (1992) proposed different analytical expressions for the momentum and energy transfer by distinguishing between sliding and non-sliding collisions of the individual particles, which are characterized by the particle-wall coefficient of friction, μ, and tangential, β_0, as well as normal, e, restitution coefficients. In order to render his calculation feasible, he restricted his theory to two limits (compare with Table 1, Eqs. (7)–(11)), where in one limit all particles slide and in the other limit all particles do not slide

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http://dx.doi.org/10.1016/j.ijmultiphaseflow.2014.12.008
0301-9322/

Table 1
Correlations of different boundary conditions for normalized shear stress and normalized flux of fluctuation energy.

Johnson and Jackson (1987)	$\frac{S}{N}=\frac{\sqrt{6}\pi}{(1+e)}\phi\, r$	(1)
	$\frac{q}{N\sqrt{3\theta}}=\frac{\sqrt{6}\pi\phi}{(1+e)}r^2+\frac{3\pi}{\sqrt{6}\pi}(1-e)$	(2)
Li and Benyahia (2012)	Revisited:	
	$\phi=(1-\frac{R}{2u_t}\frac{\partial u_t}{\partial n})\phi'$	(3)
	$\phi'=\begin{cases}-\frac{7\sqrt{6}\pi(\phi'_0)^2}{8k}r+\phi'_0 & r\leqslant\frac{4k}{7\sqrt{6}\pi\phi'_0}\\ \frac{2}{7}\frac{k}{r\sqrt{6}\pi} & \text{otherwise}\end{cases}$	(4)
	$\phi'_0=-0.0012596+0.1064551k-0.04281476k^2+0.0097594k^3-0.0012508258k^4+0.0000836983k^5-0.00000226955k^6$	(5)
	$r=u_t/\sqrt{3\Theta}, k=\frac{7}{2}\mu(1+e)$	(6)
Jenkins (1992), Jenkins and Louge (1997)	Low friction/all sliding:	(7)
	$\frac{S}{N}=\mu$	
	$\frac{q}{N\sqrt{3\theta}}=\frac{2}{(1+e)}\sqrt{\frac{2}{3\pi}}\left[\frac{1}{7}\mu_0^2-\frac{1}{2}(1-e^2)-\mu_0\mu e\left(\frac{1+e}{e+2/e_p}\right)\right]$	(8)
	Large friction/no sliding:	
	$\frac{S}{N}=\frac{3}{7}\frac{(1+\beta_0)}{(1+e)}r$	(9)
	$\frac{q}{N\sqrt{3\theta}}=-\left(\frac{\pi}{6}\right)^{1/2}\left\{\frac{2}{\pi}(1-e)+\frac{2}{7}\frac{1-\beta_0^2}{1+e}\left[\sin^4\phi_0+r^2\sin^2\phi_0\left(2\sin^4\phi_0-2\sin^2\phi_0\cos^2\phi_0-\frac{4}{(1-\beta_0)}\sin^2\phi_0\right)\right]\right.$ $+\frac{\mu}{2}\left[\pi-2\sin\phi_0\cos\phi_0+4\sin\phi_0\cos^3\phi_0-2\phi_0+r^2\left(-\pi+2\sin\phi_0\cos\phi_0+2\phi_0+16\sin^3\phi_0\cos^3\phi_0-4\sin\phi_0\cos^3\phi_0\right)\right]$ $\left.-\mu\mu_0\left[1-2\sin^2\phi_0+\sin^4\phi_0+r^2\left(2\sin^2\phi_0\cos^2\phi_0+\sin^2\phi_0\cos^4\phi_0-2\sin^4\phi_0\cos^2\phi_0\right)\right]\right\}$	(10)
	$\mu_0=\frac{7}{2}(1+e)\mu$	(11)
Schneiderbauer et al. (2012a) with $u_n = 0$	$\frac{S}{N}=\mu\, erf\left(\sqrt{\frac{3}{2}}\frac{r}{\mu_0}\right)$	(12)
	$\frac{q}{N\sqrt{3\theta}}=\sqrt{\frac{2}{\pi}}\frac{\mu^2}{\mu_0^2}(1+e-\mu_0)e^{\left(-\frac{3r^2}{2\mu_0^2}\right)}r^2$ $+\frac{\mu}{\sqrt{6\pi}\mu_0^2}(7\mu(1+e)-4\mu_0(1+\mu)-3\mu\mu_0^2(1+e))e^{\left(-\frac{3r^2}{2\mu_0^2}\right)}$ $+\frac{1}{\sqrt{6\pi}}(2(e-1)+3\mu^2(1+e))$	(13)
	$erf(x)=\frac{2}{\sqrt{\pi}}\int_0^x\exp(-\xi^2)d\xi$	(14)
	$\mu_0=\frac{7}{2}\frac{1+e}{1+\beta_0}\mu$	(15)

at impact. Based on the computer simulations of Louge (1994) Jenkins and Louge (1997) proposed revised correlations for the flux of fluctuation energy, which were still restricted to specific limits.

More recently, Li and Benyahia (2012) related the specularity coefficient to the collisional properties of friction and restitution coefficients (Table 1, Eqs. (3)–(6)). Thus, their theory distinguishes between sliding and non-sliding collisions as well. However, their work did not include a more thorough investigation of the flux of fluctuation energy.

Finally, Schneiderbauer et al. (2012a) refined the above calculations by proposing a theory, which combines sliding and non-sliding conditions in one expression for both, the solids shear stresses and the flux of fluctuation energy (compare with Table 1, Eqs. (12)–(15)).

It is noteworthy that all of the above-mentioned theories assume that the collisional granular flow is in a steady state at the wall, i.e. the granular flow can neither be in a state of compression nor in a state of expansion. Schneiderbauer et al. (2012a) suggested that this could be related to the value of the normal component of the mean solids velocity. This, in turn, implies that none of these theories can be applied to cases, where the wall moves in normal direction. To cope with this deficiency, Schneiderbauer et al. (2012a) incorporated the normal component of the solids mean velocity as well yielding a considerable impact on the shear stresses and the flux of fluctuation energy, when the granular gas is in a state of local compression or expansion. However, since all the other theories discussed here do not account for such a situation, we restrict the discussion to the case $u_n = 0$.

Finally, in Table 1 the normalized shear stress, S/N, and normalized flux of fluctuation energy, $\frac{q}{N\sqrt{3\theta}}$, as a function of normalized slip velocity, r, for the different boundary conditions, discussed above, are summarized. In Table 1, e_p is the restitution coefficient between particles. μ is wall-friction factor, ϕ and ϕ' are specularity coefficient and effective specularity coefficient, respectively. Θ denotes the granular temperature, D its dissipation and G its generation. N is the solids normal and S solids shear (=tangential) stress, R describes particle diameter, u_t and u_n the normal and tangential velocity with respect to the wall, respectively.

Discussion and evaluation of different boundary conditions

In Fig. 1 the normalized shear stress, S/N, is plotted as a function of the normalized slip velocity, $u_t/\sqrt{3\theta}$, for different values of the coefficient of wall friction, μ, and the tangential restitution coefficient, β_0. Here, the sloping part of the dotted lines corresponds to the large friction/no sliding limit of Jenkins (1992). In particular, the slope is determined by the normal and tangential restitution coefficients (Eq. (9)); the corresponding horizontal part represents the low friction/all sliding limit (Eq. (7)), which is solely given by μ. Both limits are in fairly good agreement with the discrete element simulations of Louge (1992) in the case of low and large values of r. However, at intermediate values of r the theory considerably overpredicts the normalized shear stress, S/N, since it does not include this situation appropriately (Schneiderbauer et al., 2012a).

Fig. 1(a) further demonstrates that the predictions of Johnson and Jackson (1987), i.e. the linear increase of shear stress with the slip velocity, do not obey the Coulomb limit at high slip velocities as correctly implemented by Jenkins (1992) as well as demonstrated by the data of Louge (1994). As it is obvious from Eq. (1) the