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A scaling analysis of added-mass and history forces and their coupling in dispersed multiphase flows

Y. Ling *, M. Parmar, S. Balachandar

Department of Mechanical and Aerospace Engineering, University of Florida, Gainesville, FL 32611, United States

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ABSTRACT

Accurate momentum coupling model is vital to simulation of dispersed multiphase flows. The overall force exerted on a particle is divided into four physically meaningful contributions, *i.e.*, quasi-steady, stress-gradient, added-mass, and viscous-unsteady (history) forces. Time scale analysis on the turbulent multiphase flow and the viscous-unsteady kernel shows that the integral representation of the viscous-unsteady force is required except for a narrow range of particle size around the Kolmogorov length scale when particle-to-fluid density ratio is large. Conventionally, the particle-to-fluid density ratio is used to evaluate the relative importance of the unsteady forces (stress-gradient, added-mass, and history forces) in the momentum coupling. However, it is shown from our analysis that when particle-to-fluid density ratio is large, the importance of the unsteady forces depends on the particle-to-fluid length scale ratio and not on the density ratio. Provided the particle size is comparable to the smallest fluid length scale (*i.e.*, Kolmogorov length scale for turbulence or shock thickness for shock-particle interaction) or larger, unsteady forces are important in evaluating the particle motion. Furthermore, the particle mass loading is often used to estimate the importance of the back effect of particles on the fluid. An improved estimate of backward coupling for each force contribution is established through a scaling argument. The back effects of stress-gradient and added-mass forces depend on particle volume fraction. For large particle-to-fluid density ratio, the importance of the quasi-steady force in backward coupling depends on the particle mass fraction; while that of the viscous-unsteady force is related to both particle mass and volume fractions.

1. Introduction

Modeling and simulation have become important approaches to investigate multiphase flows in engineering and environmental applications. The multiphase flows of interest here are the so-called dispersed multiphase flows, which are characterized by a dispersed phase that is distributed in a carrier phase in the form of particles, droplets, and bubbles (see Balachandar and Eaton, 2010). Examples include particle suspension in gas or liquid flows, droplet dispersion in gas flows, and bubbly flows. As can be seen in these examples, the dispersed phase can be solid, liquid, or gas; while the carrier phase can be liquid or gas. In some extreme cases, such as heterogeneous explosive detonation, the carrier phase can be solid as well (see Ling et al., 2013). For simplicity, here after we refer to the dispersed and carrier phases as "particle" and "fluid", but it should be noted that the terms "particle" and "fluid" are used in a broad sense.

In typical applications, a very large number of particles are involved and the scales of primary interest are much larger than the size of an individual particle. Therefore, it is impractical to resolve the flow details at the particle scale. Instead, the point-particle approach (PPA) is commonly used, where particles are modeled as point masses. Since the flows around the particles are not resolved, the momentum and energy coupling between fluid and particles need to be given by proper models. In the incompressible regime, Maxey and Riley (1983) and Gatignol (1983) have derived rigorous expressions for the force on a particle undergoing arbitrary time-dependent motion in an unsteady inhomogeneous ambient flow. The overall interphase coupling force can be separated into different physically meaningful contributions: the quasi-steady force $\mathbf{F}_{qs}$, the stress-gradient force $\mathbf{F}_{sg}$, the added-mass force $\mathbf{F}_{am}$, and the viscous-unsteady force $\mathbf{F}_{vu}$ (often called the Basset history force). The latter three contributions together are loosely referred as the "unsteady forces", since they are non-zero only when the acceleration of the fluid or the particle is non-zero. The Maxey-Riley-Gatignol (MRG) equation of motion has also been

* Corresponding author. Current address: Institut Jean Le Rond d'Alembert, Université Pierre et Marie Curie, 4, Place Jussieu 75252 Paris Cedex 05, France. Tel: +33 (0)1 44 27 87 14.
E-mail address: yueling@dalembert.upmc.fr (Y. Ling).

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extended to compressible flows recently by Parmar et al. (2011, 2012). The above theoretical formulations are asymptotically valid in the limit of small Reynolds and Mach numbers, but they serve as theoretical basis for empirical extension to finite Reynolds and Mach numbers.

At the level of an isolated particle in an incompressible flow, the importance of the unsteady forces compared to the quasi-steady force has been investigated by Bagchi and Balachandar (2002). Their scaling analysis showed that the ratios $|\mathbf{F}_{am}/\mathbf{F}_{qs}|$ and $|\mathbf{F}_{vu}/\mathbf{F}_{qs}|$ scale as $1/(\rho_p/\rho_f + C_M)$ and $1/\sqrt{(\rho_p/\rho_f + C_M)}$, respectively, if unsteadiness is due to particle acceleration. Here ρ_p and ρ_f are the densities of the particle and the fluid, and C_M is the added-mass coefficient. This scaling is in complete agreement with conventional expectation that in case of a particle much heavier than the fluid (*i.e.*, $\rho_p/\rho_f \gg 1$) unsteady forces are small compared to the quasi-steady force and can be ignored. This is the reason why added-mass and other unsteady forces are included in the simulations of bubbly flows (see Pougatch et al., 2008) and liquid–solid flows (see Snider et al., 1998; Patankar and Joseph, 2001) but often ignored in the context of gas-particle flows. However, this conclusion is not valid in case of unsteady effects arising from fluid acceleration. While the magnitude of particle acceleration is controlled by the particle mass, there is no such limitation to the magnitude of the ambient fluid acceleration seen by the particle. As a result, in case of fluid acceleration, Bagchi and Balachandar (2002) showed $|\mathbf{F}_{sg}/\mathbf{F}_{qs}|$ and $|\mathbf{F}_{am}/\mathbf{F}_{qs}|$ to scale as $\mathrm{Re}_p(d_p/L)$ and $|\mathbf{F}_{vu}/\mathbf{F}_{qs}|$ to scale as $\sqrt{\mathrm{Re}_p(d_p/L)}$, where Re_p is particle Reynolds number based on particle diameter (d_p) and relative velocity, and L is length scale of the ambient flow. Clearly, these ratios can be large even in case of heavier-than-fluid particles and thus when fluid acceleration is strong it may be necessary to include the unsteady forces even in case of gas–solid flows.

This situation is particularly relevant in compressible flows, where shocks and other discontinuities, as they pass over a particle, contribute to rapid variation in the ambient flow seen by the particle, see Parmar et al. (2009). The resulting unsteady force contributions arising from shock-particle interaction was systematically investigated by Ling et al. (2011a,b). Their primary conclusion was that as the shock moves over a particle the unsteady forces due to ambient fluid acceleration are an order of magnitude or more larger than the quasi-steady force. Furthermore, as can be expected from the scaling analysis of Bagchi and Balachandar (2002), the magnitude of the enhanced unsteady forces is independent of the particle-to-fluid density ratio (*i.e.*, even in case of gas–solid flows unsteady forces are large when a shock passes over the particle). However, the duration of this strong unsteady contribution is limited to only a brief period as the shock passes over the particle. Therefore, the integrated effect of the unsteady force contributions ($\mathbf{F}_{sg}$, $\mathbf{F}_{am}$ and $\mathbf{F}_{vu}$) on the long-term post-shock motion of the particle may not necessarily be significant. In terms of contribution to the long-term post-shock particle velocity, the role of unsteady force contributions was observed to be important only in case of $\rho_p/\rho_f \sim O(1)$.

The above investigations have generally been in the context of an isolated particle. As a result, they are applicable for dilute multiphase flows, where the influence of particles on the macroscale fluid motion is negligibly small. Under such dilute condition the fluid-particle interaction can be considered one-way coupled. In other words, the particle motion is dictated by the fluid flow, but the particles do not influence the fluid flow. However, in many applications, where the mass loading of particles is finite, the influence of particles on the macroscale fluid flow is significant. Then the fluid and particles are two-way coupled, see Crowe et al. (1998); Balachandar and Eaton (2010); Subramaniam (2013). In the literature, the effect of fluid on particles is usually referred as forward coupling; while the reverse effect of particles on fluid is referred backward coupling, see Garg et al. (2007) and Ling et al. (2010).

Here we are interested in multiphase flow problems where both two-way coupling and unsteady mechanisms are of importance. In this flow regime we are interested in addressing the following three fundamental questions on interphase coupling:

- Q1: *Is it possible to simplify the history integral in computing viscous-unsteady force? If so, under what conditions can this simplification be made?*
- Q2: *Under what conditions are unsteady forces important in evaluating the particle motion when compared to quasi-steady force?*
- Q3: *Under what conditions does the back effect of both quasi-steady and unsteady forces need to be taken into account in the fluid momentum equation, (i.e., under what conditions the fluid and particles are considered as two-way coupled)?*

For Q1, the viscous-unsteady force is generally computed as the Basset-like convolution integral of the past history of the relative acceleration between the particle and the surrounding fluid weighted by the history kernel. The evaluation of this convolution integral is computationally very costly. However, if the rate of change of the relative acceleration is slower than the decay of the kernel then the convolution integral can be simplified and precomputed. In this paper, by investigating the time scale of the viscous-unsteady kernel in relation to the turbulence time scale, we will establish the condition under which the convolution integral can be simplified.

For Q2, the particle-to-fluid density ratio, ρ_p/ρ_f, is conventionally used in evaluating the importance of the unsteady forces. Based on this argument, unsteady forces are often neglected in gas-particle flows, where $\rho_p/\rho_f \gg 1$. However, the scaling analysis of the unsteady forces by Bagchi and Balachandar (2002) has shown that the conventional criterion is proper in case of unsteady forces arising from particle acceleration, but must be modified if the added-mass and viscous-unsteady forces are due to fluid acceleration. Here we will extend this analysis with a more rigorous evaluation of the particle response to a range of turbulent scales. In particular, we will follow the approach of Balachandar (2009) to obtain the scales of relative velocity and relative acceleration seen by the particle in the three regimes characterized by $\tau_p < \tau_\eta$, $\tau_\eta < \tau_p < \tau_L$, and $\tau_L < \tau_p$, where τ_p is the particle response time, τ_η and τ_L are the Kolmogorov and integral time scales of the ambient flow (precise definitions of the time scales will be given in Section 3). From these characteristic scales, quantitative estimates of the relative importance of the unsteady forces in a turbulent flow will be obtained.

For Q3, the particle mass fraction ratio, defined as the mass ratio between the particles and fluid in a unit volume of the multiphase flow, is usually taken as the momentum coupling parameter, see Crowe et al. (1998). The general rule of thumb is that when the momentum coupling parameter is $O(1)$ or more, the fluid and particles are two-way coupled. This conventional momentum coupling parameter considers only the quasi-steady component of the backward coupling force, and thus may not apply when unsteady forces dominate interphase coupling. Here we will present a scaling argument to establish conditions under which quasi-steady and unsteady forces must be included in the backward momentum coupling. It should be reminded that when the volume fraction of particles is finite, the evolution of the which can also significantly influence the fluid flow through the fluid continuity equation (Eq. (1)). Nevertheless, the present paper focuses only on dilute flows where the influence of the particles on the fluid flow is only through momentum transfer.