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Nonlinear modeling of drop size distributions produced by pressure-swirl atomizers

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ABSTRACT

An axisymmetric boundary element method (BEM) has been developed to simulate atomization processes in a pressure-swirl atomizer. Annular ligaments are pinched from the parent sheet and presumed to breakup via the linear stability model due to Ponstein. Corrections to Ponstein's result are used to predict satellite droplet sizes formed during this process. The implementation provides a first-principles capability to simulate drop size distributions for low viscosity fluids. Results show reasonable agreement with measured droplet size distributions and the predicted SMD is 30–40% smaller than experiment. The model predicts a large number of very small droplets that cannot typically be resolved in an experimental observation of the spray. A quasi-3-D spray visualization is presented by tracking droplets in a Lagrangian fashion from their formation point within the ring-shaped ligaments. A complete simulation is provided for a case generating over 80,000 drops.

1. Introduction

A pressure-swirl atomizer or simplex nozzle has been widely used in a number of industries. For example, this type of atomizer is commonly used in gas turbine applications for injecting fuel into the combustor and for a number of agricultural spraying processes. A simplex nozzle generally consists of three main parts; tangential inlet ports, a swirl chamber, and an exit orifice as indicated in Fig. 1. An exit orifice is preceded by a swirl chamber with a certain contraction angle. Several inlet ports are utilized to create the vortical flow within the swirl/vortex chamber and at sufficient inlet flow velocities an air core evolves naturally within the center of this chamber. Under normal operation, the air-core extends outside the orifice creating a thin annular cone-shaped flow. The linear theory for operation of the element is well established and a variety of models exist (Lefebvre, 1989; Bayvel and Orzechowski, 1993; Yule and Chinn, 1994; Bazarov and Yang, 1998) to provide a prediction of film thickness at the nozzle exit and the resultant cone angle for the spray as a function of injector design in inflow conditions.

Scaling arguments have been made in an attempt to correlate Sauter Mean Diameter (SMD) with these parameters and as a basis for empirically based models (Rizk and Lefebvre, 1985; Suyari and Lefebvre, 1986; Lefebvre, 1989). This process requires tedious measurements at a range of flow conditions and the resultant correlations are presumed to hold only for that particular injector/atomizer geometry that was tested. More recently, stability analyses have been used to predict drop sizes by Cousin et al. (1996), Han et al. (1997) and Liao et al. (1999). Historical modeling of the spray may simply use a representative diameter (SMD) rather than a replication of the entire drop size distribution.

Higher level spray models have typically employed a variety of distribution functions (Rosin Rammler, Nukiyama–Tanasawa, etc.) to provide an analytic description of the spray that requires only a few inputs from experimental measurements. Unfortunately, simulations for many engineering problems display a high degree of sensitivity to the overall distribution within the spray thereby placing substantial requirements on the modeler to improve the distribution functions to the greatest extent possible. For example, in combustion problems, the ignition kinetics are dominated by the smallest drops while the overall combustion time/efficiency is chiefly determined by the largest drops, i.e., the tails of the distribution function are terribly important to the overall modeling result.

For these reasons, there is a strong motivation to develop spray distributions from first principles such that the vagaries of the fitting of the distribution function are no longer an issue. The ever-increasing computational power now affords modelers a limited capability to conduct such simulations and that serves as the motivation for the subject work.

The simplex nozzle is an ideal testbed for the development of analytic/computational spray distributions because these injectors produce a reasonably axisymmetric swirling sheet that serves as the initial condition for atomization events. For modest injection speeds, ligaments with significant azimuthal extent are observed to be pinched from the periphery of the conical sheet (Kim et al.,

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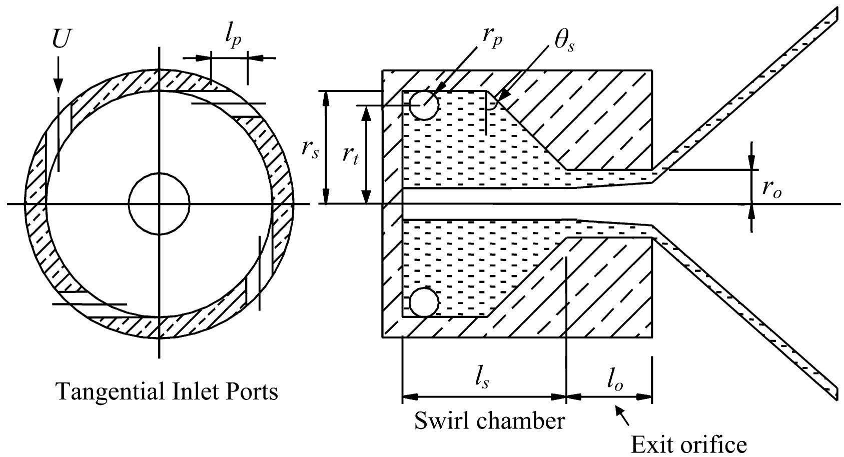

Fig. 1. Geometrical characteristics of simplex nozzle.

2003). At higher injection velocities, turbulent and three-dimensional instabilities, amplified by aerodynamic interactions with the gas, lead to reductions in the azimuthal extent of these ligaments; in the limit direct pinching of droplets from the conical sheet can be observed (Dumouchel, 2008). For this reason, the model described herein will be most applicable to moderate injection speeds wherein sectors of ring-shaped ligaments are formed, and those ligaments subsequently fractionate into drops. This assumption provides for a drastic reduction in computational expense such that full spray simulations are achievable within current computational resources.

The boundary element method (BEM) has been previously applied to investigate instability of free surface and atomization of jets (Hilbing and Heister, 1996, 1998; Heister et al., 1997; Heister, 1997; Rump and Heister, 1998; Yoon and Heister, 2004; Park et al., 2006; Park and Heister, 2006). Park (Park et al., 2006; Park and Heister, 2006) and Yoon (Yoon and Heister, 2004; Park et al., 2006) are credited with development of the elements of the model utilized in the present work. While the BEM formulation is inherently inviscid, a treatment to handle weak viscous effects due to Lundgren and Mansour (1988) has been implemented (Park and Heister, 2006) to provide some consideration for viscous stresses within the fluid.

The BEM has substantial capabilities to simulate highly nonlinear capillary flows including pinching events because all the nodes are placed on the surface of interest. The objective of the present work is to develop simplex nozzle spray distributions from first principles. The model is briefly described in the following section, followed by grid convergence studies, comparisons with measured data, and conclusions from the study.

2. Model description

2.1. Boundary element method

A brief description of the model is provided here in the interest of brevity. Yoon and Heister (2004) provide a complete description of the basic model elements and Park and Heister (2006) provide a detailed description of the incorporation of swirling effects. Under the assumption of inviscid, incompressible, and axisymmetric flow, the flow dynamics are governed by Laplace's equation, $\nabla^2\phi = 0$. Laplace's equation is transformed to the integral form (Yoon and Heister, 2004) as follows:

$$\alpha\phi(\vec{r}_i) + \int_\Gamma \left[\phi\frac{\partial G}{\partial \hat{n}} - qG\right] d\Gamma = 0 \tag{1}$$

where $\phi(\vec{r}_i)$ is the value of the velocity potential at a point $\vec{r}_i$, q is the differentiation of ϕ with respect to normal vector $\hat{n}$, Γ is the boundary of the domain, α is the singular contribution resulting from integration over the "base point" in question, and G is the free space Green's function for the Laplacian operator. Following Liggett and Liu (1983), the free space Green's function for the axisymmetric Laplacian can be expressed in terms of elliptical integrals of the first and second kinds. Under the assumption that the velocity potential ϕ and the normal velocity q vary linearly over the length of an element, the governing equation yields a linear system of equations relating local ϕ and q values.

The unsteady Bernoulli equation provides a boundary condition at the free surface and relates capillary, hydrostatic, centrifugal, and dynamic pressure forces to the local surface shape. For the swirling flow, modifications are required to account for the pressure gradient created by the swirl. Swirling flows are considered via a superposition of a potential vortex with the base flowfield. If we choose the liquid density (ρ), inlet jet velocity (U), and radius of orifice (r_o) as dimensions, the nondimensional unsteady Bernoulli equation can be written (note that the superscript * is omitted here):

$$\frac{D\phi}{Dt} = \frac{1}{2}|\nabla\phi|^2 - P_g - \frac{\kappa}{\mathrm{We}} + \frac{\mathrm{Bo}}{\mathrm{We}}z \tag{2}$$

with nondimensional parameters as follows:

$$\nabla\phi^* = \frac{\nabla\phi}{U} \quad \phi^* = \frac{\phi}{Ur_o} \quad P_g^* = \frac{P_g}{\rho U^2} \quad t^* = \frac{U}{r_o}t \tag{3}$$

where P_g is dimensionless gas pressure, κ is the local surface curvature and the Weber number ($\mathrm{We} = \rho U^2 r_o/\sigma$) and Bond number ($\mathrm{Bo} = \rho g r_o^2/\sigma$) appear as governing dimensionless parameters. The total surface velocity $\vec{u}_t$ is computed via a superposition of a potential vortex (ϕ_v, $\vec{u}_v$) with the base axial flow (ϕ, $\vec{u}$). The velocity potential and axial, radial, and circumferential velocity components (u, v, w) can be written as follows:

$$\phi_t = \phi + \phi_v \quad u_t = u + u_v \quad v_t = v + v_v \quad w_t = w + w_v \tag{4}$$

Upon superposition of a potential vortex, combining Eqs. (2) and (4) yields the dimensionless unsteady Bernoulli equation:

$$\frac{D\phi}{Dt} = \frac{1}{2}|\vec{u}_t|^2 - \vec{u}_t\cdot\vec{u}_v - P_g - \frac{\kappa}{\mathrm{We}} + \frac{\mathrm{Bo}}{\mathrm{We}}z \tag{5}$$

The velocity components of the vortex are as follows:

$$u_v = 0 \quad v_v = 0 \quad w_v = \frac{\Gamma_v}{2\pi r} \tag{6}$$