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Contact forces between viscoelastic ellipsoidal particles

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ABSTRACT

The normal (NFD) and tangential contact forces (TFD) between viscoelastic ellipsoidal particles are studied based on the contact mechanics and finite element method. It is found that the NFD and TFD force models previously formulated for spheres are valid for ellipsoidal particles and non-spherical particles of smooth surface, provided that some variables in the models are properly considered. The applicability of the NFD and TFD models are demonstrated in the collisions of two viscoelastic ellipsoids and shown to agree well with the results calculated by means of the finite element method. As part of the study, the applicability of the Linear-Spring-Dashpot (LSD) model which is widely used in the discrete modeling is also examined, and its limitation is identified.

1. Introduction

The Discrete Element Method (DEM) introduced by Cundall and Strack [1] has become a popular tool to study the dynamics of granular materials. Compared with continuum-based methods, DEM can avoid artificial assumptions about the constitutive and other closure relations and provide useful microscopic information such as the trajectory of and contact forces acting on an individual particle. It treats granular materials as a system of discrete soft particles where the motion of a particle is controlled by the interactions from its surrounding particles or fluid according to the Newton's second law of motion [2]. However, to produce reliable simulation results, the inter-particle contact forces must be accurately determined.

The studies of the contact mechanics between solids date back to Hertz [3] for the normal force and Mindlin–Deresiewicz (MD) [4,5] for the tangential force. The two classical theories consider ideal elastic–frictional solids, hence known to be insufficient to represent the real contact properties, especially in terms of energy dissipation. Inelastic behaviors such as elastoplasticity and viscoelasticity are therefore generally considered to be more accurate in deriving contact laws. For example, Thornton [6], Vu-Quoc et al. [7] and Li et al. [8] have respectively proposed different NFD models for elastic–perfectly plastic spheres. The tangential force of elastoplastic spheres was also investigated by Vu-Quoc et al. [9] by use of the finite element method (FEM) and resulted in a quite complicated TFD model. Wu et al. [10–12] analyzed the rebound behavior of elastoplastic spheres with a wide range of impact angles and tested their results against experimental and FEM results. The contact mechanics between viscoelastic spheres have also been studied by other investigators based on the Hertz and MD theories [13–15]. Supported by the FEM results, a recent study of the contact of a viscoelastic sphere with a rigid wall suggests a set of semi-theoretical force models for DEM simulations [16,17].

However, all those studies focused on ideal spheres which are just an approximation to engineering particles. In practice, particle shape varies and is reported to affect granular packing and flow significantly [18–23]. There are increasing uses of non-spherical particles, e.g., ellipsoids [18–21] and polygons [22,23], in DEM simulations of granular materials. Nevertheless, previous studies are mainly concerned with the algorithm of contact detection. How to precisely evaluate the contact forces between non-spherical particles is still lack of serious investigation, although some studies have been conducted on the force treatment of non-spheres comprised by 'glued spheres' [24,25].

In this work, we present an FEM study of NFD and TFD characteristics of viscoelastic ellipsoids. The theoretical aspects of contact mechanics in the normal and tangential directions are described in Section 2, where a set of semi-theoretical models are derived in connection with our previous study [16]. The comparison of the models with FEM results is given in Section 3 together with a detailed description of the FEM simulation. The effectiveness of the viscoelastic NFD and TFD models is examined in different collisions between two ellipsoids. Finally, the main findings from this study are summarized in Section 4.

2. Theoretical treatments

2.1. Normal contact force

2.1.1. The Hertz theory

Prior to the study of viscoelastic contact, it is helpful to give a brief summary of the elastic contact theory of Hertz which is the basis for

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the present analysis. The Hertz theory applies to general solids provided that the surfaces of solids are continuous and the deformation of solids induced by contact is far smaller than the dimensions of each solid [26]. Due to the localized nature of most contact problems, the deforming regions of contact surfaces are very small and can be approximated as quadric curved surfaces. That is, regardless of the whole shapes of solids, their contact properties are mainly determined by the local geometries in the vicinity of the contact point, characterized by two principal radii of curvature R_1', R_1'', and R_2', R_2'', as well as the corresponding principal axes, $\mathbf{T}_1'$, $\mathbf{T}_1''$ and $\mathbf{T}_2'$, $\mathbf{T}_2''$ as shown in Fig. 1. In most cases, the principal axes of two surfaces do not coincide but are inclined to each other by an angle α. When loaded in the normal direction, the two solids deform and touch each other over an elliptical area with major and minor semi-axes being a and b respectively. For spheres, the ellipse reduces to a circle with $a = b$. For the contact shown in Fig. 1, a summary of the contact quantities are given as follows.

$$\text{Normal force}\quad F_{\text{Hertz}} = \frac{4}{3}E^*\sqrt{R_e}\delta_n^{3/2}F_2^{-3/2} \tag{1}$$

$$\text{Equivalent radius of contact area}\quad \sqrt{ab} = R_e\delta_n\frac{F_1^2}{F_2} \tag{2}$$

$$\text{Eccentricity of contact ellipse}\quad e^2 = 1-\left(\frac{b}{a}\right)^2 \approx 1-\left(\frac{A}{B}\right)^{4/3} \tag{3}$$

where F_{Hertz} is the normal contact force according to the Hertz theory, δ_n is the relative displacement of two particles in the normal direction, $E^* = \{(1-v_1^2)/E_1 + (1-v_2^2)/E_2\}^{-1}$, where E and v are respectively Young's modulus and the Poisson ratio, and $R_e = (AB)^{-1/2}/2$ is the effective radius. F_1 and F_2 are the correction factors which equal unity for circular contact. Their expressions are quite complex as described in [26]. In practice, approximate equations are often used. For example, below is the one formulated by Hale [27]:

$$F_1 = 1-\left[\left(\frac{B}{A}\right)^{0.0602}-1\right]^{1.456} \quad F_2 = 1-\left[\left(\frac{B}{A}\right)^{0.0684}-1\right]^{1.531} \tag{4}$$

where variables A and B are determined from the local geometrical variables by the following equations [26]:

$$\begin{aligned} A+B &= \frac{1}{2}\left(\frac{1}{R_{1'}}+\frac{1}{R_{1''}}+\frac{1}{R_{2'}}+\frac{1}{R_{2''}}\right) \\ B-A &= \frac{1}{2}\left\{\left(\frac{1}{R_{1'}}-\frac{1}{R_{1''}}\right)^2+\left(\frac{1}{R_{2'}}-\frac{1}{R_{2''}}\right)^2+2\left(\frac{1}{R_{1'}}-\frac{1}{R_{1''}}\right)\left(\frac{1}{R_{2'}}-\frac{1}{R_{2''}}\right)\cos 2\alpha\right\}^{1/2}. \end{aligned} \tag{5}$$

Once the principal radii of curvature R_1', R_1'', and R_2', R_2'', and the inclined angle α are known, one can determine the values of A and B from Eq. (5), the correction factors F_1 and F_2 from Eq. (4) and the effective radius according to the definition $R_e = (AB)^{-1/2}/2$. Then the normal contact force F_n can be calculated according to Eq. (1). The profile of the contact area, determined by parameters a and b, can be obtained by solving Eqs. (2) and (3). Generally, the calculation of the principal radii of curvature is complex for non-spherical particles. We show how to do so for the ellipsoids in Appendix A.

2.1.2. Viscoelastic NFD

Consistent with the previous studies [13–17], in this work, the viscoelastic behavior is defined by the following constitutive relations:

$$\begin{aligned} \sigma_{ij} &= \sigma_{ij}^e + \sigma_{ij}^v \\ \sigma_{ij}^e &= E_1\left(\varepsilon_{ij}-\frac{1}{3}\delta_{ij}\varepsilon_{kk}\right)+E_2\varepsilon_{kk}\delta_{ij} \\ \sigma_{ij}^v &= \eta_1\left(\dot{\varepsilon}_{ij}-\frac{1}{3}\delta_{ij}\dot{\varepsilon}_{kk}\right)+\eta_2\dot{\varepsilon}_{kk}\delta_{ij} \end{aligned} \tag{6}$$

where $E_1 = E/(1+v)$ and $E_2 = E/3(1-2v)$ are elastic constants, and η_1 and η_2 are coefficients of viscosity related to shear and bulk deformation, respectively. δ_{ij} is the Kronecker symbol. An additional viscous stress σ_{ij}^v is introduced in case of viscoelastic material which is proportional to the rate of strain. Similar to the treatment of the spheres in the previous studies [14,16], this viscous stress is expressed as

$$\sigma_{ij}^v = \dot{\delta}_n\frac{\partial}{\partial\delta_n}\left[\eta_1\left(\varepsilon_{ij}-\frac{1}{3}\delta_{ij}\varepsilon_{kk}\right)+\eta_2\varepsilon_{kk}\delta_{ij}\right]. \tag{7}$$

Generally, the strain field ε_{ij} in Eq. (7) is unknown in advance. Hertzsch et al. [28] and Brilliantov et al. [14] proposed a quasistatic approximation to overcome this problem, assuming that the displacement velocities in the bulk are much smaller than the wave speed in

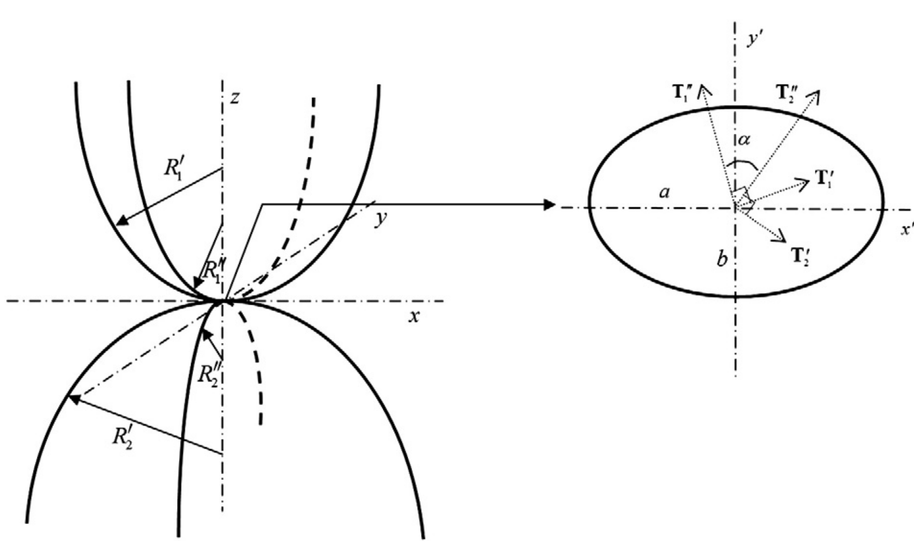

Fig. 1. The local surfaces in the vicinity of contact position. After ref. [44].