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Reducing the number of size classes in a cumulative rates model used for process control of a grinding mill circuit

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ABSTRACT

The number of size classes in a cumulative rates model of a grinding mill circuit is reduced to determine the minimum number required to provide a reasonably accurate model of the circuit for process control. Each reduced size class set is used to create a non-linear cumulative rates model which is linearized to design a linear model predictive controller. The accuracy of a model is determined by the ability of the corresponding model predictive controller to control important process variables in the grinding mill circuit as represented by the full non-linear cumulative rates model.

Results show that a model with 25 size classes that provides valuable information for plant design and scale-up, can be reduced to a model containing only a small number of size class sets and still be suitable for process control. Although as few as 3 size classes can be used to obtain a fairly accurate model for process control, the distribution of these 3 size classes influences the accuracy of the model. For a model to be useful for process control, the model should at least provide the directions in which the process variables change.

1. Introduction

When modeling a grinding mill circuit with a population balance model, the question arises: how many and which size classes should be used to characterize the material in the circuit? The studies of [2] and [3] use 27 size classes in their models, the model of [18] uses only 3 size classes and the model of [7] uses only 2 size classes to characterize ore. The latter two models were developed for process control purposes and the model in [18] has been used in a robust nonlinear model predictive controller for a grinding mill circuit [11]. The advantage of a model with fewer size classes is that it is simpler to incorporate in a model-based controller scheme. However, a model-based controller is dependent on an accurate and reliable process model. Therefore, it is of interest to determine the minimum number of size classes that yields a good model for process control purposes.

It can be argued that a model should have a minimum of four or five size classes. The size classes should account for the usually bimodal size distributions of run-of-mine ore feed, mill discharge and hold-up. The size classes should therefore reflect the following:

- Slimes from zero to a few microns that have transport behavior that follows that of water. The model of [3] assumes that the specific discharge rate function is constant up to about 1 mm.
- Fines from a few microns to around 13–25 mm that should obey normal breakage behavior and would be the typical feed for a conventional ball mill.
- Critical size material from 13–25 mm to 50–100 mm that exhibits abnormal breakage behavior. This material does not self-break, is difficult to grind and is inefficiently broken by coarse rock and steel grinding media. This is particularly true for fully-autogenous (FAG) and semi-autogenous (SAG) mills.
- Rocks coarser than about 100 mm that self-breaks to form pebble grinding media, but leads to critical size problems.

This article investigates how many size classes are necessary to accurately simulate a grinding mill circuit with a cumulative rates model for process control purposes. The study uses the sampling campaign data of an optimisation study of an industrial circuit treating Merensky ore [14]. A base set of 25 size classes is reduced to smaller sets and each reduced set is used to model the circuit. Since the cumulative rates model is non-linear, each model is linearized before it is used to design a linear model-based controller. The controllers are implemented on a grinding mill circuit simulated by the non-linear model with 25 size classes. The controller performance is used as a measure of the accuracy with which a size class set models the circuit. Because the controllers are model-based, the performance also indicates the least number of size classes that should be included in a model for process control.

2. Grinding mill circuit description

According to the survey by [29] ball mills are most common in industrial plants, followed closely by FAG and SAG mills. A ball mill is usually fed with crushed ore, whereas SAG and FAG mills are fed with ROM ore. In FAG mills the grinding media consist only of ore, whereas ball and SAG mills are fed with steel balls to assist with ore

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breakage [28]. A ROM ore SAG mill with a high ball load and an end-discharge screen in a single-stage closed circuit configuration, as shown in Fig. 1, is considered for this study.

The three main elements in Fig. 1 are the mill, the sump and the hydrocyclone. The mill receives four streams: mined ore (*MFS*), water (*MIW*), steel balls and underflow from the hydrocyclone. The ground ore in the mill mixes with the water to create slurry. The fraction of the mill volume filled with charge is represented by *JT*. The slurry is then discharged to the sump. The slurry from the mill can be discharged either by overflow or through an end-discharge screen. In the case of the screen, the particle size of the discharged slurry from the mill is limited by the aperture size of the screen. The slurry level in the sump is represented by *SLEV*. The slurry in the sump is diluted with water (*SFW*) before it is pumped to the cyclone for classification. The outflow of the pump is the classifier feed flow *CFF*. The hydrocyclone is responsible for the separation of the in-specification and out-of-specification ore discharged from the sump. The lighter, smaller and in-specification particles of the slurry pass to the overflow of the hydrocyclone, while the heavier, larger and out-of-specification particles pass to the underflow. The underflow is passed to the mill for further grinding. The overflow contains the final product, measured in terms of the fraction of particles smaller than the specification size (*PSE*), that is passed to a downstream process [11,28].

A description of the variables in Fig. 1 and their respective units can be found in Table 1. These variables are commonly controlled and manipulated in grinding mill circuits [29].

3. The cumulative rates model

The cumulative rates model in [16] is a simple population balance model based on the assumption that only one function is necessary to describe grinding kinetics inside the mill, as illustrated in [7]. This function is the cumulative breakage rate function, which is defined as the rate per unit mass that a given species coarser than a given size breaks to below that size [6]. This assumption gives an advantage over the population balance model of [30] and [3] which requires two functions, the breakage rate and the appearance function, to describe the grinding kinetics. The parameters of the breakage function in the cumulative breakage rate model can be back calculated from plant measurements [16].

The cumulative rates model has been successfully used to simulate a SAG grinding mill circuit in [1,2]. A dynamic simulator based on the cumulative rates model was coupled to an on-line parameter estimator and validated in simulation with industrial plant data in [26]. The cumulative rates model was also used by [17] and [14] to improve the operating performance of industrial grinding mill circuits. A discussion of the application of this model to SAG mills can be found in [16].

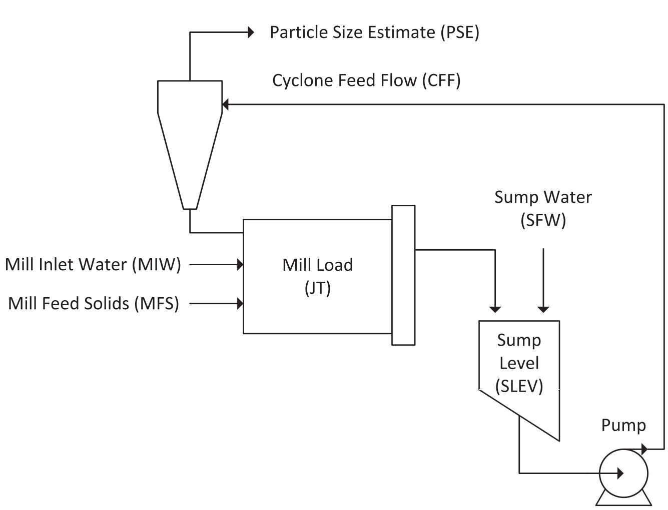

Fig. 1. A single-stage closed grinding mill circuit where the manipulated variables are *MIW*, *MFS*, *SFW*, and *CFF*, and the controlled variables are *PSE*, *SLEV* and *JT*.

Table 1
Description of circuit variables.

Variable	Description
Manipulated variables	
CFF	Flow-rate of slurry to the cyclone [m^3/h]
MFS	Feed-rate of ore to the mill [t/h]
MIW	Flow-rate of water to the mill [m^3/h]
SFW	Flow-rate of water to the sump [m^3/h]
Controlled variables	
JT	Fractional volumetric filling of mill [%]
PSE	Product particle size estimate [fraction $\leq$ 75 μm]
SLEV	Slurry level in sump [m]

A drawback in the cumulative breakage rate model is the assumption that the cumulative rates of breakage of ore above a given size x_i is unaffected by the grinding environment and the structure of the size distribution above x_i. This drawback also holds for the parameters of the breakage rate and appearance function of the models of [30] and [3]. If grinding conditions depart significantly from those used to derive the cumulative breakage rate parameters, it is possible to get negative flow rates in some of the size classes — especially the smaller size classes. This implies that the parameters need to be adjusted according to the changes in the grinding conditions. Including the effect of grinding environment changes on the cumulative breakage rate function is a possible improvement of the model, but does not form part of this study. It can be assumed that for a SAG mill the values for the cumulative breakage rate function remain fairly constant as long as the ball filling and internal charge level remain fairly constant [1].

3.1. Cumulative rates model description

The circuit shown in Fig. 1, with variables described in Table 1, is considered in this study. The cumulative rates model of [16], as it defines the mill, the sump and the hydrocyclone in Fig. 1, is described here. The nomenclature of the model can be seen in Table 2.

3.1.1. Mill model

For the cumulative rates model particle sizes are reported as the cumulative percentage of ore smaller than size x_i (mm). The first and largest size x_1 is selected to be larger than the largest particles likely to be encountered in the feed stream. The sink size class n represents all particles with sizes between zero and x_n [16].

The specific cumulative breakage rate function K_i is the fractional rate at which particles above a given size x_i in the mill break to below that size per unit time. The energy-normalized cumulative breakage rate function is given by $K_i^E = (M/P)K_i$ and is generally insensitive to scale-up [1]. M (t) is the ore hold-up in the mill and P (kW) is the net mill power. The unit for K_i^E is [kWh/t]$^{-1}$.

The mill model considers a continuously fed mill and treats it as a single fully mixed reactor. The population balance equation with respect to time for size class 1 is:

$$\frac{dw_i}{dt} = f_1 - g_1 w_1 - w_1 \frac{P}{M} K_2^E \quad (1)$$

where w_1 (t) is the absolute mass retained in size class 1, f_1 (t/h) is the absolute mass flow-rate for the mill feed for size class 1 and g_1 (h^{-1}) is the specific discharge rate for size class 1. Since there are no particles larger than x_1, K_1^E is undefined and K_2^E is used in Eq. (1) to describe the breakage of ore in size class 1 to smaller sizes.