Powder Technology 246 (2013) 625–634

Contents lists available at SciVerse ScienceDirect

Powder Technology

journal homepage: www.elsevier.com/locate/powtec

Optimum choice of the make-up ball sizes for maximum throughput in tumbling ball mills ☆

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ARTICLE INFO

Article history:
Received 20 February 2013
Received in revised form 8 May 2013
Accepted 19 June 2013
Available online 27 June 2013

Keywords:
Make-up balls
Ball diameter
Grinding circuit simulation
Ball wear model

ABSTRACT

A grinding circuit simulation combined with ball weal law was used to determine the optimum composition of the make-up ball sizes in tumbling ball mills. It was found that the optimum composition depends on various factors, including the feed size, the product size, the mill diameter and the breakage parameters. In all cases, binary mixtures of two ball sizes (50.8 mm and 25.4 mm) performed better than a mixture of the three ball sizes. An equation therefore could be developed for calculating the optimum composition of the make-up balls as a function of various parameters.

1. Introduction

The ball size is one of the critical factors for determining the mill performance of ball mills. It is well known that larger balls are needed for the effective breakage of large-size particles, whereas smaller balls are more effective for the breakage of fine particles. Therefore, it is a common practice in industry to use a mixture of balls rather than balls of a single-size to ensure the efficient grinding of materials of various sizes in the mill. Various formulae have been proposed for the selection of the ball size [1,2]. However, as a group, they are not entirely satisfactory because the optimum mixture of balls depends on the feed size as well as the product size. Further, the ball size distribution in the mill is not a simple parameter that can be controlled directly, as it depends on the make-up ball charge and wear rate. Therefore, the industrial practice of determining the make-up ball sizes comes down to experience. However, there is a great deal of information that describes the variation of the grinding kinetics with the ball size and the ball wear kinetics. It is possible then to incorporate this information into a grinding model and investigate the effect of the make-up ball size on the mill performance with various feed and product specifications.

Concha et al. [3] was the first to combine a grinding circuit model with a ball wear model to optimize the make-up ball charge. With an optimization algorithm, the optimum make-up was calculated to perform a given task. However, their work was based on one set of milling conditions, one set of a feed size distribution and breakage characteristics, and one mill diameter. All of these variables are potentially significant factors influencing the optimum choice of make-up balls. In this study, a more comprehensive investigation was conducted to delineate the effect of variables, specifically the breakage parameters, the feed size, the product size, and the mill diameter, via a grinding circuit simulation combined with a ball wear model.

2. Theory

2.1. Kinetic grinding model

A fundamental understanding of the breakage process was developed using the size–mass balance or population balance approach analogous to the chemical reactor design for first-order reactions. This approach is based on experimental batch grinding results which typically demonstrate that the rate of the breakage of a material of a size within a $\sqrt{2}$ sieve interval follows a first-order breakage law. Symbolically, this is expressed as

$$\frac{dw_i(t)}{dt} = -S_i w_i(t) \tag{1}$$

where S_i is the specific rate of the breakage of size i, and $w_i(t)$ is the mass reaction of size i at time t. Therefore, S is the equivalent of a first-order chemical rate constant.

Fig. 1 shows the typical variation of the S values with particle size x_i for a given ball diameter. It can be seen that the S values increase

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http://dx.doi.org/10.1016/j.powtec.2013.06.026

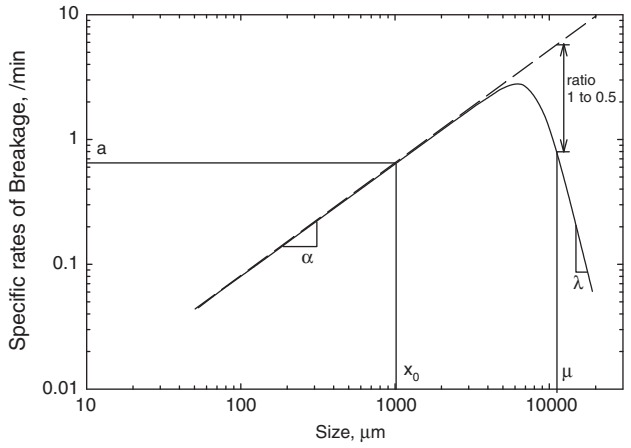

Fig. 1. Variation of the specific rates of breakage with particle sizes.

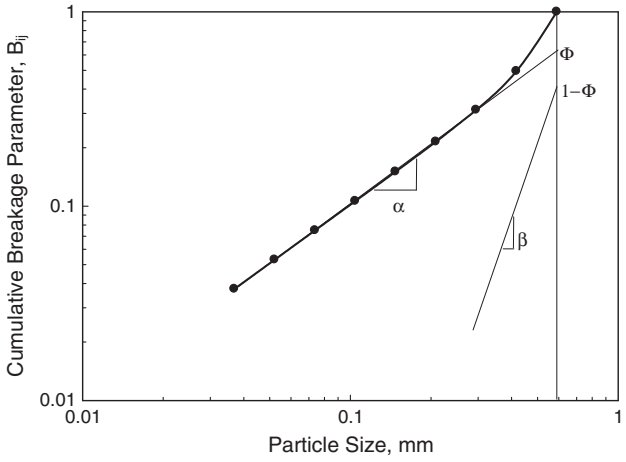

Fig. 2. Cumulative breakage distribution and its characteristic parameters.

with size, but reach a maximum and then decrease as the particles become too large to be broken efficiently by the grinding media. This relationship can be fitted empirically to an equation which consists of two functions [4]:

$$S_i = A\left(\frac{x_i}{x_o}\right)^{\alpha} \frac{1}{1+\left(\frac{x}{\mu}\right)^{\Lambda}}. \tag{2}$$

The first part of the equation is a power function which gives a straight line on a log–log scale. α is the slope of the curve in the smaller region and A is the S value for the standard size, x_o. The second part of the equation is a log-logistic function representing deviation from the straight line relationship. It has a value of 1 for smaller sizes and approaches 0 as the size becomes very large. μ is the particle size at which the value is 0.5 and Λ is a positive number which is an index of how rapidly the S values fall as the particle size increases.

The breakage of a uniformly sized particle results in the production of an entire set of smaller product sizes, thus requiring a description for this distribution of sizes. The mean set of sizes produced by primary breakage before re-fracture occurs is termed the primary breakage distribution, b_{ij}, defined as the weight fraction of broken products from size interval j which appears in size interval i. The breakage distribution is often used in the cumulative form, $B_{ij} = \sum_{k=n}^{i} b_{kj}$ which is the cumulative weight fraction of material broken from size interval j which appears less than the upper size of size interval i.

The values of the primary breakage distribution are often found to be insensitive to the milling conditions such as the media and powder loading. For some materials, the curves of B_{ij} fall on top of one another for all values of j when B_{ij} is plotted versus the relative size x_i/x_j. This is termed the 'normalized' B_{ij}, and it means that all of the particles break into a fragment distribution with dimensional similarity; that is, the weight fraction of the product less than, for example, half of the breakage size is constant for all breaking sizes.

Austin and Luckie [5] describe a mathematical technique for characterizing the typical breakage distribution, as shown in Fig. 2. The values of B_{ij} are fitted by an empirical function made up of the sum of two power functions [4]:

$$B_{ij} = \varnothing\left(\frac{x_{i-1}}{x_j}\right)^{\gamma} + (1-\varnothing)\left(\frac{x_{i-1}}{x_j}\right)^{\beta}. \tag{3}$$

Here, γ is the slope of the small size end of the distribution, $\varnothing$ is the extrapolated intercept of this end, and β is the slope of the upper part of the curve, as depicted in Fig. 2. These parameters depend on the characteristics of the material being ground, but are often found to be independent of the milling conditions.

2.2. Grinding circuit simulation

The size reduction process can be described by a general size–mass balance equation through the transfer function, d_{ij}, as follows [4]:

$$p_i = \sum_{j=1}^{i} d_{ij} f_j \qquad i<j<1. \tag{4}$$

In this equation, d_{ij} is the weight fraction of the feed of size j transferred by breakage to product of size i, p_i is the weight fraction of the product of size i and f_j is the weight fraction of the feed of size j. For continuous milling, residence time distribution models can be incorporated into the d_{ij} values, taking the following form [4]:

$$d_{ij} = \begin{cases} e_j, & i=j \\ \sum_{k=j}^{i-1} c_{ij}c_{jk}\left(e_k - e_j\right), & i>j \end{cases} \tag{5}$$

$$c_{ij} = \begin{cases} -\sum_{k=i}^{i-1} c_{ij}c_{jk}, & i<j \\ 1, & i=j \\ \frac{1}{S_i - S_j}\sum_{k=j}^{i-1} S_k b_{ik} c_{kj}, & i>j \end{cases} \tag{5a}$$

$$e_{j=}\int_0^{\infty} e^{-S_j t}\psi(t)dt. \tag{5b}$$

Here, $\psi(t)$ is the residence time distribution function. It is convenient to represent the RTD in a functional form. Various forms have been used to describe the RTD of grinding mills. This includes the single fully mixed, the m equal fully mixed model and the axial mixing model [4]. Among these models, the one-large/two-small fully mixed reactor-in-series model was found to fit all the data reasonably well [6]. This model gives

$$e_j = \frac{1}{\left(1+S_j\tau_1\right)\left(1+S_j\tau_2\right)^2} \tag{6}$$

where τ_1 is the mean residence of the first reactor, and τ_2 is the mean residence time of the second and third reactor having an equal size.

This equation allows the calculation of the product size distribution from a mill once d_{ij} is determined. The calculation of d_{ij} includes the aforementioned S and B functions, which can be determined in a laboratory batch mill. However, these parameters are sensitive to milling conditions such as the mill rotational speed, ball filling,