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Low-Reynolds-number motion of a deformable drop between two parallel plane walls

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Abstract

The motion of a three-dimensional deformable drop between two parallel plane walls in a low-Reynolds-number Poiseuille flow is examined using a boundary-integral algorithm that employs the Green's function for the domain between two infinite plane walls, which incorporates the wall effects without discretization of the walls. We have developed an economical calculation scheme that allows long-time dynamical simulations, so that both transient and steady-state shapes and velocities are obtained. Results are presented for neutrally buoyant drops having various viscosity, size, deformability, and channel position. For nearly spherical drops, the decrease in translational velocity relative to the undisturbed fluid velocity at the drop center increases with drop size, proximity of the drop to one or both walls, and drop-to-medium viscosity ratio. When deformable drops are initially placed off the centerline of flow, lateral migration towards the channel center is observed, where the drops obtain steady shapes and translational velocities for subcritical capillary numbers. With increasing capillary number, the drops become more deformed and have larger steady velocities due to larger drop-to-wall clearances. Non-monotonic behavior for the lateral migration velocities with increasing viscosity ratio is observed. Simulation results for large drops with non-deformed spherical diameters exceeding the channel height are also presented.

Keywords: Drops; Bubbles; Channel; Boundary-integral; Stokes flow

1. Introduction

Droplet motion in confined geometries, aside from being of fundamental interest, is relevant to several application areas, ranging from multiphase fluid flow through porous media to the transport of cells or other biological media in capillaries and microfluidic devices. In the present work, the behavior of a single deformable drop in a Poiseuille flow between two infinite parallel plane walls is examined by numerical simulation, focusing primarily on cases when the drop size is comparable to the channel height, i.e. when the bounding walls affect the drop shape and mobility.

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The motion of spherical or nearly spherical drops in a channel consisting of two parallel walls has received attention by a variety of exact and approximate methods. The parallel motion of a nearly spherical drop between two channel walls in a quiescent fluid was considered by Shapira and Haber (1988) using the method of reflections. Approximate solutions for the hydrodynamic drag force exerted on the droplet were obtained, which are accurate when the drop-to-wall spacing is not small. More recently, Chen and Keh (2001) utilized a boundary-collocation technique to examine the parallel motion of spherical drops moving near one plane wall and between two parallel plates as a function of drop size and viscosity ratio. The solutions of Chen and Keh (2001) agree well with a previous study on the motion of rigid spheres (Ganatos et al., 1980) when the drop-to-medium viscosity ratio tends to infinity. The motion of rigid particles in Stokes flow between two planar walls has also been studied (Staben et al., 2003), where a boundary-integral method was used to find the translational and rotational velocities of spherical and ellipsoidal particles, as functions of particle size and location in the channel.

In comparison to numerical investigations involving rigid particles, there are two primary factors arising for deformable drops that introduce enriched physical phenomena at low-Reynolds number. First, one must consider the dynamically coupled flows exterior and interior to the drop interface. Second, the viscous stresses exerted on the drops by the flow give rise to deformations. In contrast to spherical drops and particles, a neutrally buoyant drop in fully developed channel flow is capable of crossing streamlines, as a consequence of drop deformation. Under such conditions, the drops can obtain a variety of transient non-spherical shapes, requiring dynamical simulations and an accompanying increased computational demand compared to non-deformable drops or rigid particles.

The lateral migration of two-dimensional drops in a channel consisting of two parallel plane walls has been studied numerically for finite Reynolds number (Mortazavi and Tryggvasson, 2000), with the full Navier–Stokes equations solved by a second-order projection method, using a finite-difference/front-tracking approach to examine dynamical drop behavior, primarily as a function of the Reynolds number. Although the study was focused towards higher Reynolds numbers and smaller relative drop sizes and deformations than those in the present work, some small inertia ($Re = 0.25$) cases for neutrally buoyant drops were considered. Of particular interest, when deformable drops with diameter 1/4 of the channel height were initially placed off the centerline of flow, movement towards the centerline for a low-viscosity drop was reported, while migration away from the centerline was observed for a drop with viscosity matching the external fluid. The motion of a two-dimensional bubble rising in an inclined channel, with and without insoluble surfactant, has also received attention by a boundary-integral method (DeBisschop et al., 2002).

Also related to the problem at hand is the motion of deformable drops through cylindrical tubes, which has received considerable attention and is motivated by several applications in the field of biomechanics. For example, the motion of red blood cells through veins or capillaries, as well as the fate of gas bubbles in the blood stream, is of significant biological and clinical interest. Olbricht and Kung (1992) have experimentally studied the motion of drops in straight tubes for an extensive range of parameters. The axisymmetric motion of deformable drops in pressure-driven flow has been considered by Martinez and Udell (1990). The deformation of axisymmetric drops and bubbles moving through straight tubes and constrictions under pressure-driven flow have been studied by Tsai and Miksis (1994) as a function of capillary number. Using a combination of lubrication theory for the thin film between the drop and the tube wall and a two-dimensional boundary-integral representation for the internal flow, Hodges et al. (2004) recently considered the motion of a semi-infinite drop moving through a cylindrical tube. Such approaches are not a replacement for the numerical simulations in our work, since they are valid for over a limited range of parameters (e.g. very small-drop-to-wall clearances and capillary numbers) and have only logarithmic accuracy. Coulliette and Pozrikidis (1998) considered the transient motion of a periodic file of three-dimensional drops in a cylindrical tube by numerical simulation for subcritical capillary numbers. In their study, the drop-to-medium viscosity and density ratios were fixed at unity and the surface tension was assumed constant. The principal objective of then analysis was to extend previous two-dimensional and axisymmetric studies of drop motion by examining the dynamics of droplet migration, when the drops are initially placed off the centerline of flow. Assuming the capillary number is sufficiently large, the drops begin to deform from their initially spherical shape, migrate towards the centerline of flow, and then approach a steady shape after a preliminary stage of rapid deformation. The results given by Coulliette and Pozrikidis (1998) provide some