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Application of response surface methodology for modeling of ball mills in copper sulphide ore grinding

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ABSTRACT

Modeling of some parameters of wet ball milling system of copper sulphide ore was performed in this study. A three level Box–Bhenken design combining a response surface methodology (RSM) with quadratic model was employed for modeling of key operating parameters of ball mills. Grinding experiments were designed and executed by a laboratory ball mill, considering ball size, ball charge and solid content as variables. Grinding tests were performed changing these three variables (ball size, ball charge and solid content) in the range of 20–40 mm, 20–40% and 65–80% respectively. Product 80% passing size (d80) was defined as process response. A quadratic model was developed to demonstrate the effect of each parameter and its interaction with d80 of product. Predicted values of response obtained using model equation were in good agreement with the experimental values (R^2 value of 0.994 for d80). Finer d80 was achieved using greater ball charges with smaller ball sizes. More favorable results were also obtained at the center of solid content level. Results suggest that RSM could be efficiently applied for modeling of ball milling system of some copper sulphide ores.

1. Introduction

In today's global markets suffering from the world crisis, mining groups strive to optimize mill performances mainly by reducing production costs. Successful grinding with ball mills depends on selecting suitable operating conditions. Therefore it is of great importance to determine the operating parameters at which the response reaches its optimum.

Different variables have been studied to improve ball mill performance [1–3]. The slurry density and grinding media size are probably the most frequently considered factors for the process optimization [3]. The efficiency of grinding depends on the surface area of the grinding medium. Thus, balls should be as small as possible and the charge should be graded in such manner that the largest balls are just heavy enough to grind the largest and hardest particles in the feed [4]. Harder ores and coarser feeds require high impact and large media while very fine ground sizes require substantial media surface area and small media [5].

The pulp density of the feed should be as high as possible, consistent with ease of flow through the mill. It is essential that the balls are coated with a layer of ore; too dilute a pulp increases metal-to-metal contact, giving increased steel consumption and reduced efficiency. Also slurry density certainly influences the distribution of the energy of impacts applied to the particles in a grinding mill [6]. The energy input to a mill is also of great importance and valuable works are focused on this area [7,8]. It increases with the ball charge, and reaches a maximum at a charge volume of approximately 50%, but for a number of reasons, 40–50% is rarely exceeded [4].

As an essential approach it is very economical to adopt an experimental design methodology for extracting the maximum amount of complex information while saving significant experimental time, material used for analysis and personnel costs [9]. Engineering experimentalists wish to find the conditions under which a certain process is optimized. The optimum could be either a maximum or a minimum of a function of the design parameters. One of the methodologies for obtaining the optimum is the response surface technique [10].

A large number of studies have been conducted trying to apply different types of modelling methods to a relatively wide range of process optimization [11–14].

Experimental design methods and response surface methodologies have been specifically applied for modeling of process parameters in mineral processing systems [15–21]. Response surface methodology (RSM) has been employed for modeling of some processes such as Turkish coal processing [16], flotation of celestite concentrate [17], chromite concentration [18], flotation of synthetic mixture of celestite and calcite minerals [19] and sulphur grindability in a batch ball mill [20]. RSM also has been applied for optimization of yield at a desired ash level in coal flotation [21].

To optimize the milling process, influencing factors such as reduction ratio or d80 of product were considered to be closely monitored.

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In current study, grinding efficiency improvement is investigated by making key changes to milling system as follows:

- Using correct ball size, operating the mill in an appropriate mode (ball charge and solid content) and controlling milling circuit in a modified mode.

Reduction ratio is a determining factor in mill efficiency evaluation which can show how efficiently the energy is consumed. A higher reduction ratio can signal a more efficient milling in progress.

In this study RSM was used in conjunction with Box–Bhenken design, which requires fewer tests than a full factorial design, to establish the functional relationship between the three operating variables of grinding (ball size, ball charge and solid content) and d_{80} of product (response) for copper sulphide grinding. These relationships can by that means be used to determine the optimal operating parameters. Subsequently, the application of RSM and Box–Bhenken design to the modeling of the influence of three operating variables on the ball mill performance for grinding of copper sulphide is discussed.

2. Materials and methods

2.1. RSM

RSM is a collection of statistical and mathematical techniques useful for the modeling and analysis of problems in which a response of interest is influenced by several variables and the objective is to optimize this response [22]. In most RSM problems, the form of the relationship between dependent and the set of independent variables is unknown. Thus, the first step in RSM is to find a suitable approximation for a functional relationship between dependent and independent variables. Second-order models are widely used in RSM as they have several advantages. They are very flexible and can take on a wide variety of functional forms so they will work well as an approximation to the true response surface [22–24].

The design procedure of RSM is as follows [14,16,25]:

I. Designing of a series of experiments for adequate and reliable measurement of the response of interest.
II. Developing a mathematical model of the second-order response surface with the best fittings.
III. Finding the optimal set of experimental parameters that produce a maximum or minimum value of response.
IV. Representing the direct and interactive effects of process parameters through two and three-dimensional (3D) plots.

If all variables are assumed to be measurable, the response surface can be expressed as follows:

$$y = f(x_1, x_2, x_3, \ldots, x_k) \tag{1}$$

Where y is the answer of the system, x_i ($i = 1-k$) is the variable of action called factor and k is the number of variables. The goal is to optimize the response variable (y).

2.2. Box–Bhenken design

It is assumed that the independent variables are continuous and controllable by experiments with negligible errors. It is required to find a suitable approximation for the true functional relationship between independent variables and the response surface. These variables were changed during the tests with respect to the Box–Bhenken experimental design, whereas the other operational parameters of grinding were kept constant (feed amount, feed size, grinding time and mill speed).

Usually a second-order model is utilized in RSM (Eq. (2)) [9,10,25].

$$y = \beta_0 + \sum_{i=1}^{k} \beta_i x_i + \sum_{i=1}^{k} \beta_{ii} x_i^2 + \sum_{i=1}^{k-1} \sum_{j=2}^{k} \beta_{ij} x_i x_j + \varepsilon \tag{2}$$

Where $x_1, x_2, \ldots, x_k$ are the input factors which influence the response y; β_0, β_{ii} ($i = 1,2,\ldots,k$), β_{ij} ($i = 1,2, \ldots, k$; $j = 1,2, \ldots, k$) are unknown parameters and ε is a random error. The β coefficients, which should be determined in the second-order model, are obtained by the least square method. In general, Eq. (2) can be written in matrix form as Eq. (3):

$$Y = bX + \varepsilon \tag{3}$$

Where Y is defined to be a matrix of measured values, X to be a matrix of independent variables. The matrixes b and ε consist of coefficients and errors, respectively. The solution of Eq. (3) can be obtained by the matrix approach.

$$b = \left(X'X\right)^{-1} X'Y \tag{4}$$

Where X' is the transpose of the matrix X and $(X'X)^{-1}$ is the inverse of the matrix $X'X$.

2.3. Materials and experimental procedure

For this study, materials (copper sulphide ore) were sampled from the SAG mill feed in Sungun copper concentrator plant. The density of ore was 2.8 g/cm^3. The ore was crushed in a laboratory scale jaw crusher and roll crusher successively to prepare materials for ball mill grinding tests. Mineralogical composition of the ore sample was also analyzed using XRD and the results were presented in Table 1.

The size distribution of ore prepared for ball mill feed was shown in Fig. 1. As shown in Fig. 1, the d80 of sample (as ball mill feed) was 480 μm. The measured Bond work index of ore was 16 that obtained from Bond grindability test.

Batch grinding tests were carried out using a 25.8*20.8 cm (length*diameter) ball mill equipped with 4 lifters. The L/D ratio in both laboratory ball mill and the plant ball mill was similar.

The maximum ball size calculated from Bond formula for plant ball mill (Eq. (5)) [11] was 30 mm.

$$d_B(mm) = 25.4\left[\left(\frac{F_{80}}{k}\right)^{0.5}\left(\frac{s.g.W_i}{100C_s\sqrt{3.281D}}\right)^{0.33}\right] \tag{5}$$

Where F_{80}: feed size 80% passing (μm), s.g.: specific gravity of ore feed, W_i: feed ball mill work index (kWh/t), D: the inside diameter of the mill (m), C_s: critical speed (Cs) is 13.5 rpm, k: a constant designated as the mill factor (350).

To apply an average size of 30 mm, ball size range was considered to be in a range of 20–40 mm.

Table 1
Mineralogical composition of the feed ore sample using XRD.

Component	Weight (%)
Cu_2S	0.391
CuS	0.284
$CuFeS_2$	0.431
FeS_2	6.628
MoS_2	0.037
Fe_2O_3–$Fe(OH)_2$	0.140
Fe_2O_3	0.189
Fe_3O_4	0.105
SiO_2	67.27
Al_2O_3	14.38